```
> with(inttrans) :
> with(student, changevar) :
> with(plots);
```

[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot] **(1)**

```
> alias(u = u(x, t), U = U(k, t))
```

Point, u, U **(2)**

```
> eq := diff(u, t) - c^2 diff(u, x, x) = 0
```

$$eq := \frac{\partial}{\partial t} u - c^2 \left(\frac{\partial^2}{\partial x^2} u \right) = 0 \quad \textbf{(3)}$$

```
> eq1 := subs(fourier(u, x, k) = U, fourier(eq, x, k));
```

$$eq1 := c^2 k^2 U + \frac{\partial}{\partial t} U = 0 \quad \textbf{(4)}$$

```
> dsolve(eq1, U);
```

$$U = \_F1(k)\, e^{-c^2 k^2 t} \quad \textbf{(5)}$$

```
> subs(_F1(k) = F(k), %)
```

$$U = F(k)\, e^{-c^2 k^2 t} \quad \textbf{(6)}$$

```
> Su := u = invfourier(rhs(%), k, x)
```

$$Su := u = invfourier\left(F(k)\, e^{-c^2 k^2 t}, k, x\right) \quad \textbf{(7)}$$

```
> convert(Su, int);
```

$$u = \frac{1}{2} \frac{\int_{-\infty}^{\infty} F(k)\, e^{-c^2 k^2 t + I k x}\, dk}{\pi} \quad \textbf{(8)}$$

```
> assume(c > 0); assume(k > 0); assume(t > 0); Su := 1/(2 pi) Int(Int(f(xi)·exp(-c^2·k^2·t
    - I·k·xi + I·k·x), k = -infinity..infinity), xi = -infinity..infinity);
```

$$Su := \frac{1}{2} \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)\, e^{-c\sim^2 k\sim^2 t\sim - I k\sim \xi + I k\sim x}\, dk\sim\, d\xi}{\pi} \quad \textbf{(9)}$$

```
> innerint := int(exp(-c^2·k^2·t - I·k·xi + I·k·x), k = -infinity..infinity);
```

(10)

$$innerint := \frac{e^{-\frac{1}{4}\frac{(-\xi + x)^2}{c\sim^2\, t\sim}}\sqrt{\pi}}{c\sim\sqrt{t\sim}} \quad \textbf{(10)}$$

> $Su := simplify\left(Int\left(f(\mathrm{xi})\cdot simplify\left(\frac{innerint}{2\cdot \mathrm{Pi}}\right), \mathrm{xi} = -infinity..infinity\right)\right);$

$$Su := \int_{-\infty}^{\infty} \frac{1}{2}\,\frac{f(\xi)\, e^{-\frac{1}{4}\frac{(-\xi + x)^2}{c\sim^2\, t\sim}}}{c\sim\sqrt{t\sim}\,\sqrt{\pi}}\, \mathrm{d}\xi \quad \textbf{(11)}$$

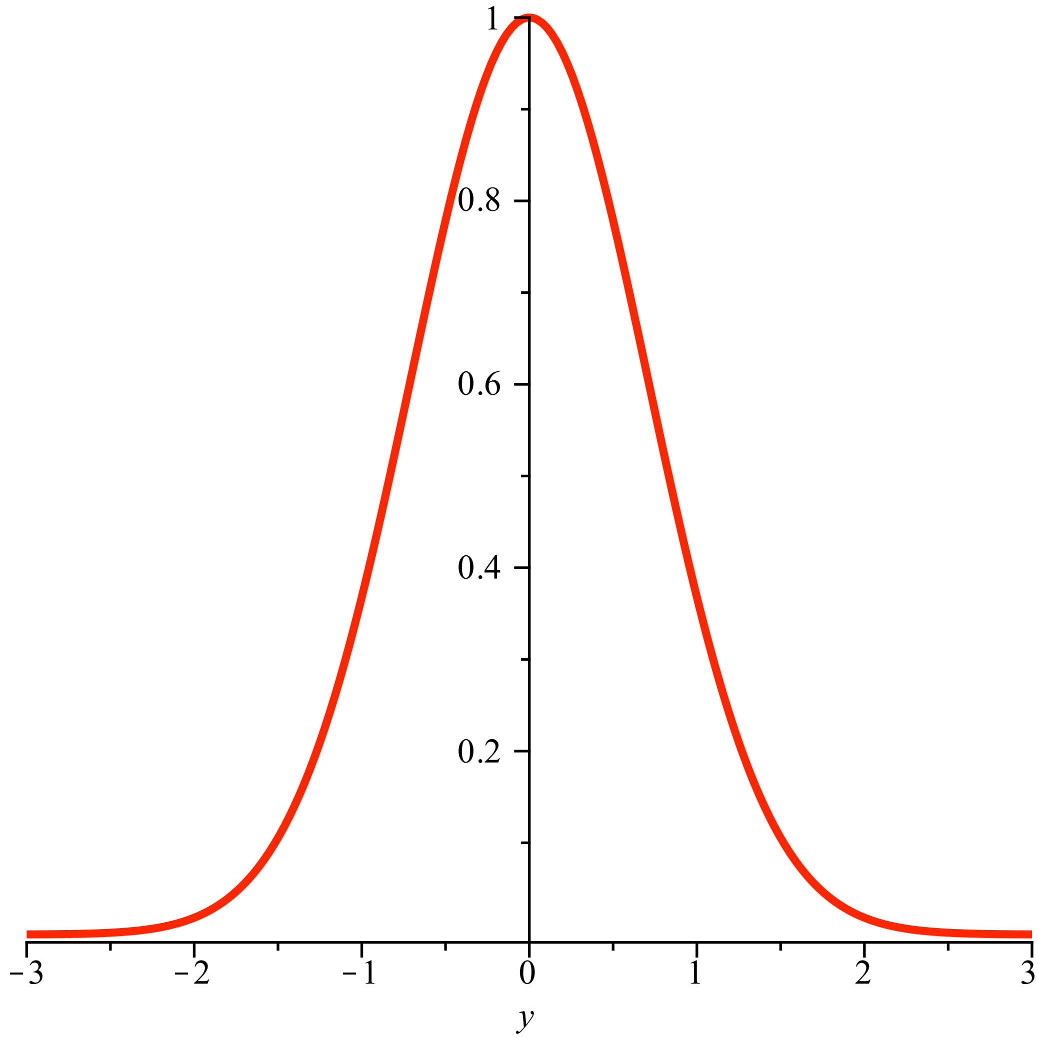
> $u = \%$

$$u(x, t\sim) = \int_{-\infty}^{\infty} \frac{1}{2}\,\frac{f(\xi)\, e^{-\frac{1}{4}\frac{(-\xi + x)^2}{c\sim^2\, t\sim}}}{c\sim\sqrt{t\sim}\,\sqrt{\pi}}\, \mathrm{d}\xi \quad \textbf{(12)}$$

> $f1 := y \rightarrow \exp(-y^2);\ assume(t, positive);\ assume(c, positive);$

$$f1 := y \rightarrow e^{-y^2} \quad \textbf{(13)}$$

> $plot(f1(y), y = -3..3, thickness = 3, color = red);$



> $Su1 := simplify(value(subs(f = f1, Su)));$

$$Su1 := \frac{e^{-\frac{x^2}{4\,c\sim^2\,t\sim+1}}}{\sqrt{4\,c\sim^2\,t\sim+1}} \tag{14}$$

> $u := (x, t) \rightarrow \frac{\exp\left(-\frac{x^2}{(4\cdot c^2\cdot t+1)}\right)}{(4\cdot c^2\cdot t+1)^{\left(\frac{1}{2}\right)}}$

$$u(x, t\sim) := (x, t) \rightarrow \frac{e^{-\frac{x^2}{4\,c^2\,t+1}}}{\sqrt{4\,c^2\,t+1}} \tag{15}$$

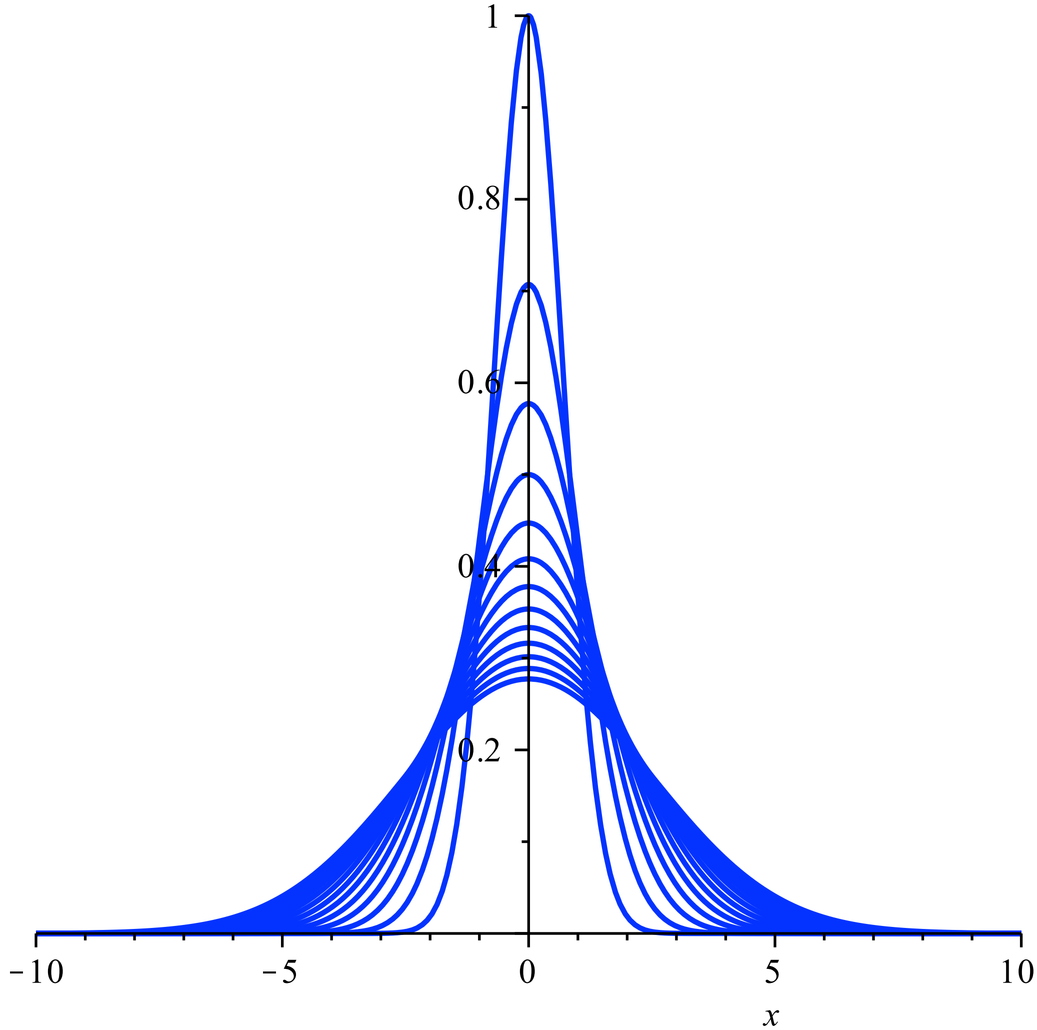
> $c := \frac{1}{2}$

$$c := \frac{1}{2} \tag{16}$$

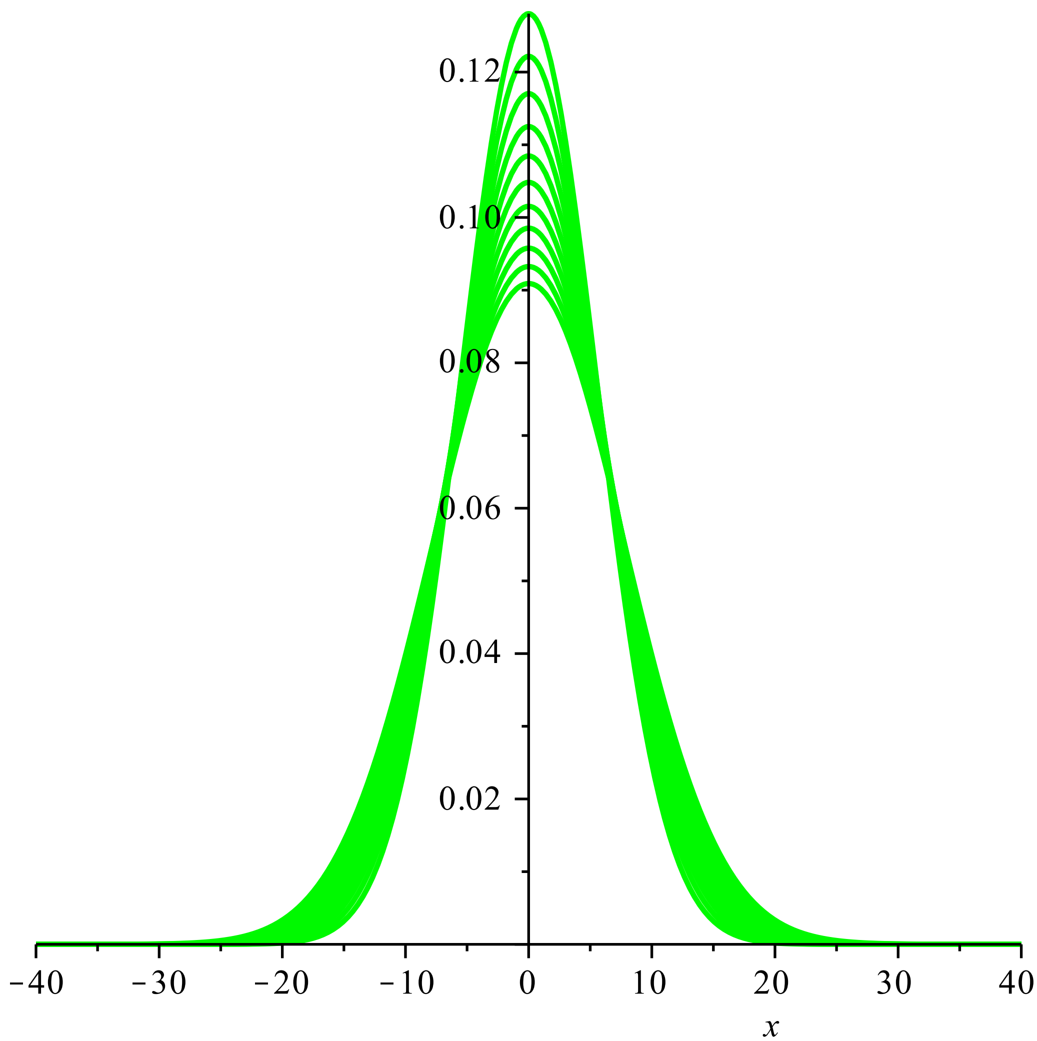
> $p1 := seq(plot(u(x, i), x = -10..10, color = blue, thickness = 2), i = 0..12)$:

> $p2 := seq(plot(u(x, 6\cdot i), x = -40..40, color = green, thickness = 2), i = 10..20)$:

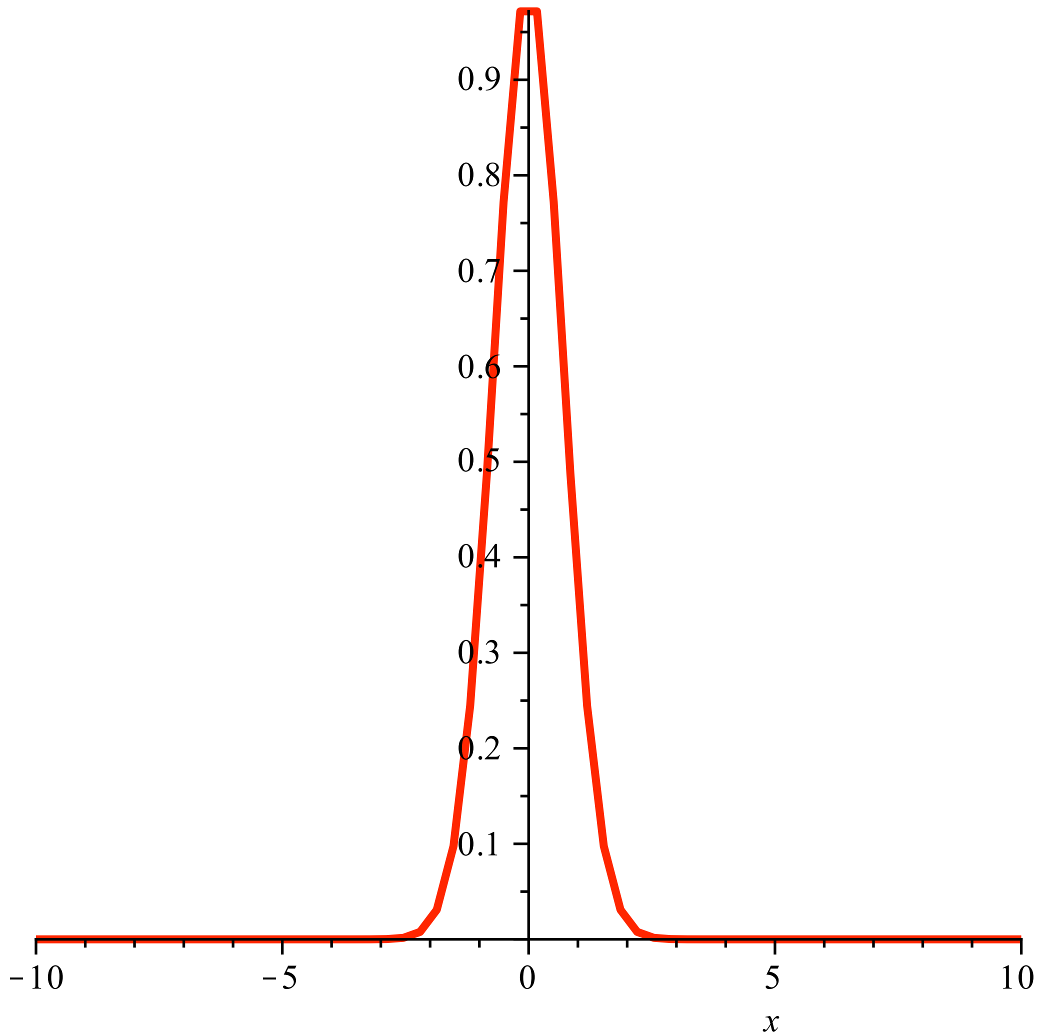
> $display([p1])$;



> $display([p2])$;



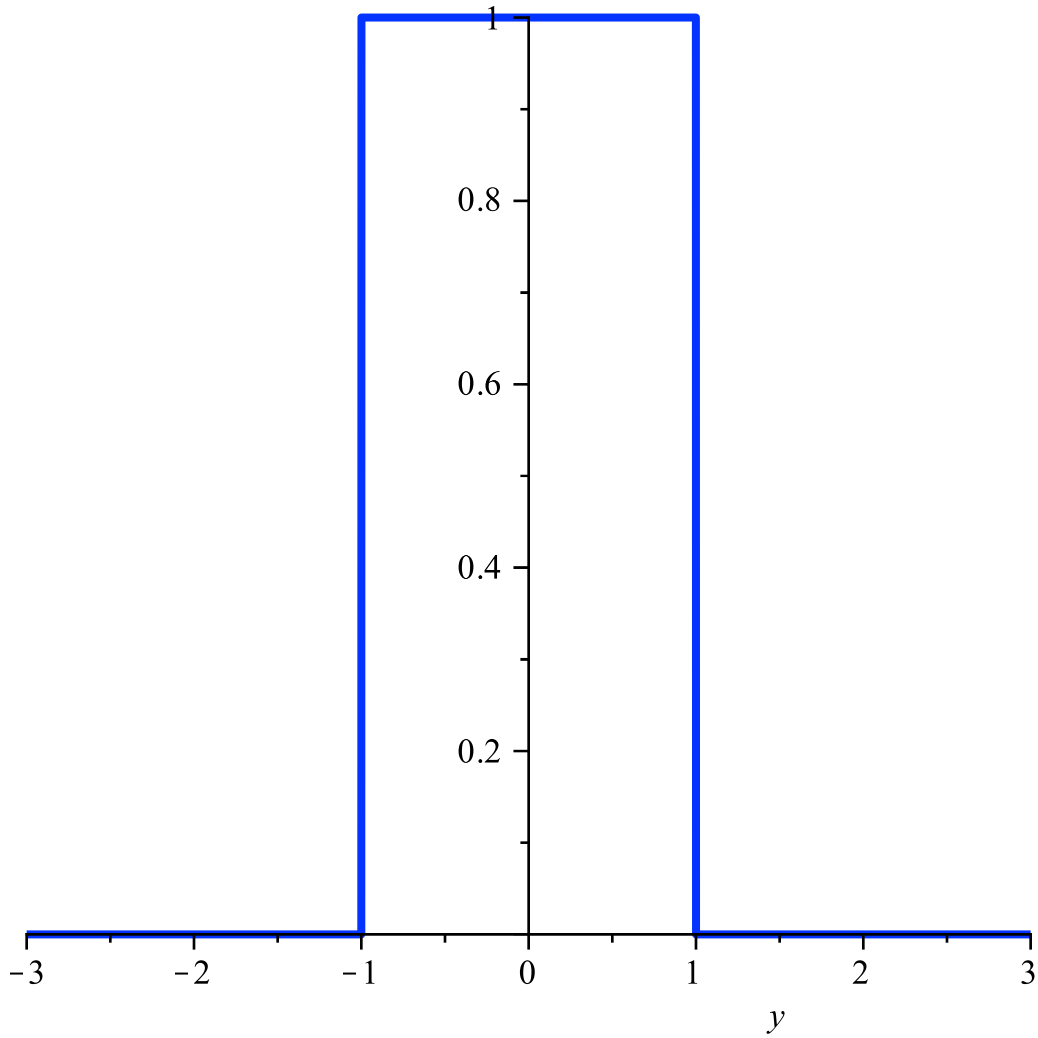
> $animate(u(x, i), x = -10..10, i = 0..10, thickness = 3, numpoints = 60)$



> $f2 := y \to \text{Heaviside}(y+1) - \text{Heaviside}(y-1)$

$$f2 := y \to \text{Heaviside}(y+1) - \text{Heaviside}(y-1) \quad \textbf{(17)}$$

> $plot(f2(y), y = -3..3, thickness = 3, color = blue)$



> $Su2 := simplify(value(subs(f=f2, Su)));$

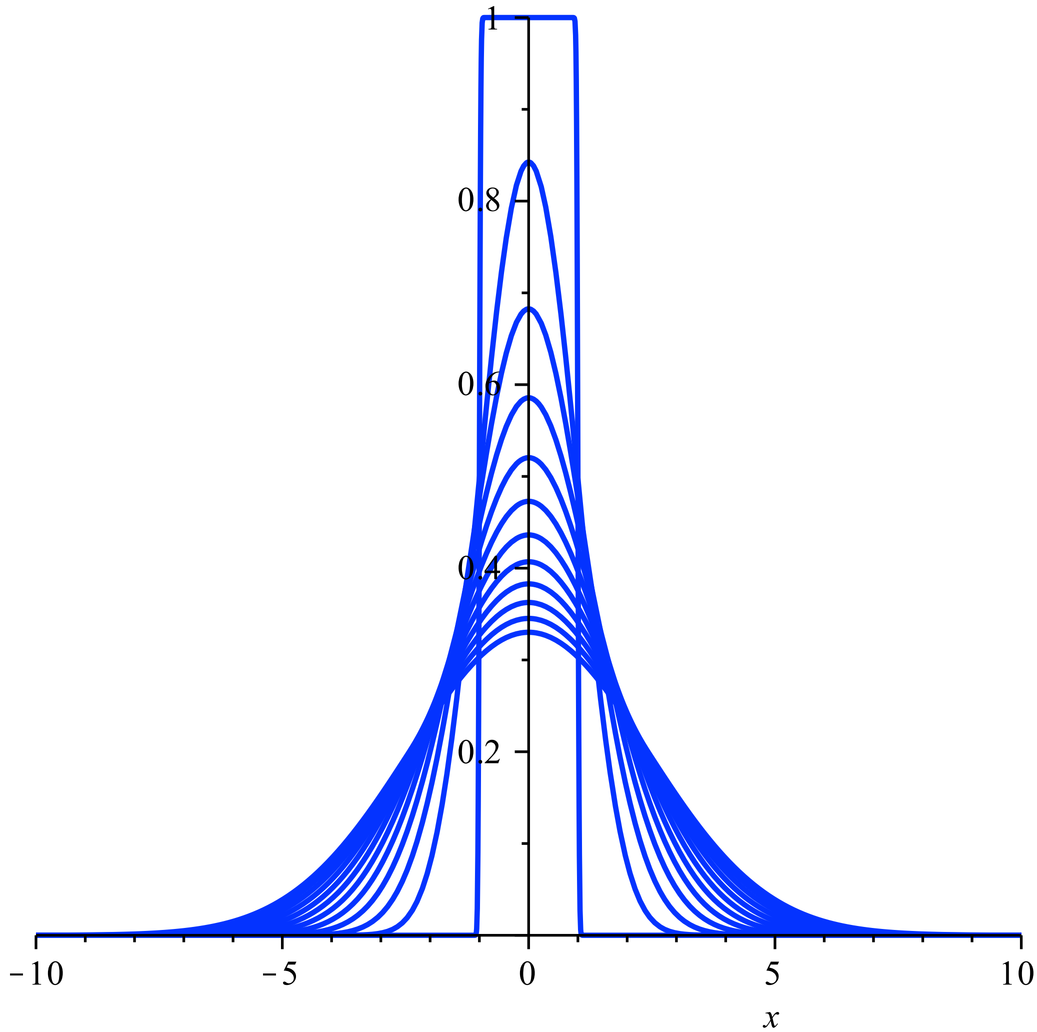
$$Su2 := -\frac{1}{2}\,\mathrm{erf}\left(\frac{1}{2}\,\frac{-1+x}{c\sim\sqrt{t\sim}}\right)+\frac{1}{2}\,\mathrm{erf}\left(\frac{1}{2}\,\frac{1+x}{c\sim\sqrt{t\sim}}\right) \quad \textbf{(18)}$$

> $u2 := (x,t) \rightarrow -\frac{1}{2}\mathrm{erf}\left(\frac{1}{2}\,\frac{-1+x}{c\cdot \mathrm{sqrt}(t)}\right)+\frac{1}{2}\mathrm{erf}\left(\frac{1}{2}\,\frac{1+x}{c\cdot \mathrm{sqrt}(t)}\right)$

$$u2 := (x,t) \rightarrow -\frac{1}{2}\,\mathrm{erf}\left(\frac{1}{2}\,\frac{x-1}{c\sqrt{t}}\right)+\frac{1}{2}\,\mathrm{erf}\left(\frac{1}{2}\,\frac{x+1}{c\sqrt{t}}\right) \quad \textbf{(19)}$$

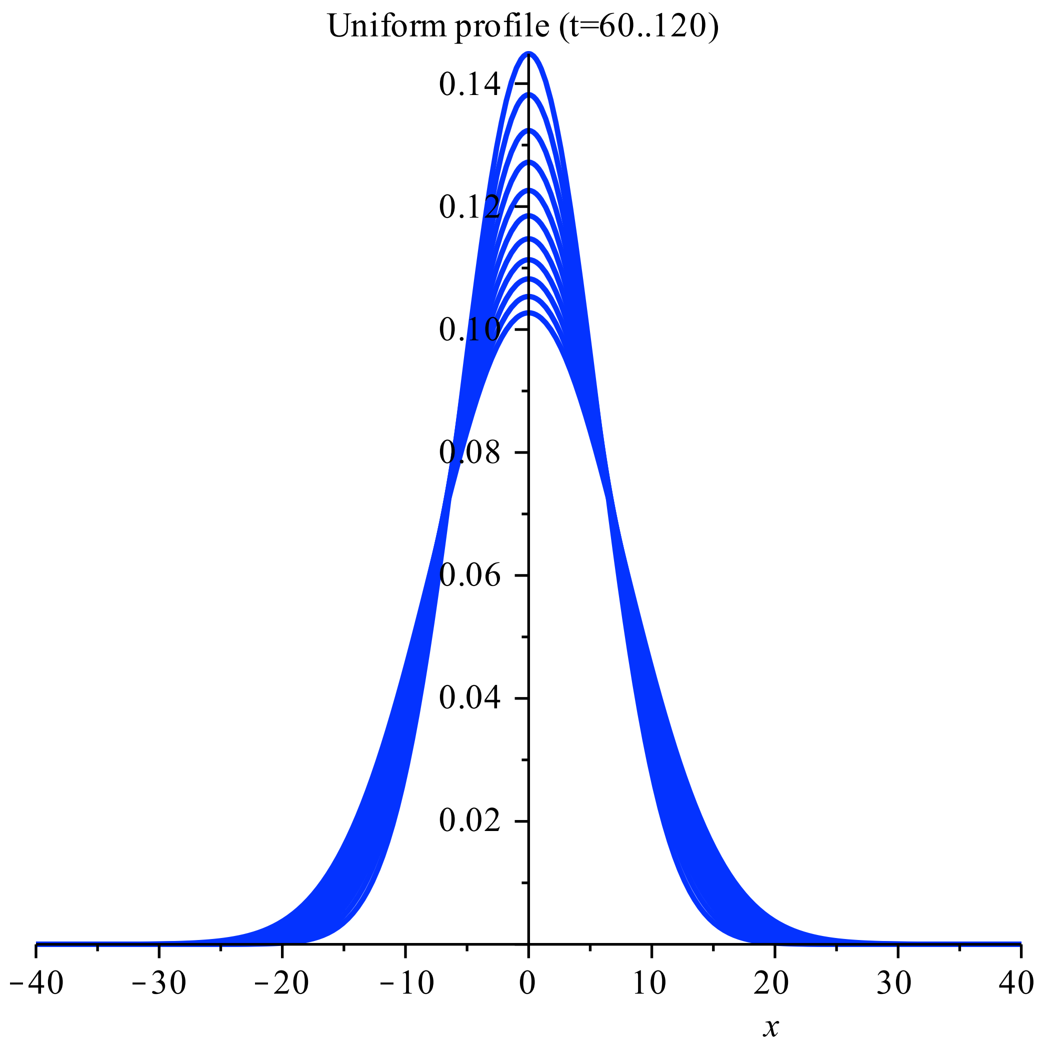
> $p3 := seq(plot(u2(x, i), x = -10 ..10, color = blue, thickness = 2), i = 0.001 ..12):$

> $display([p3])$

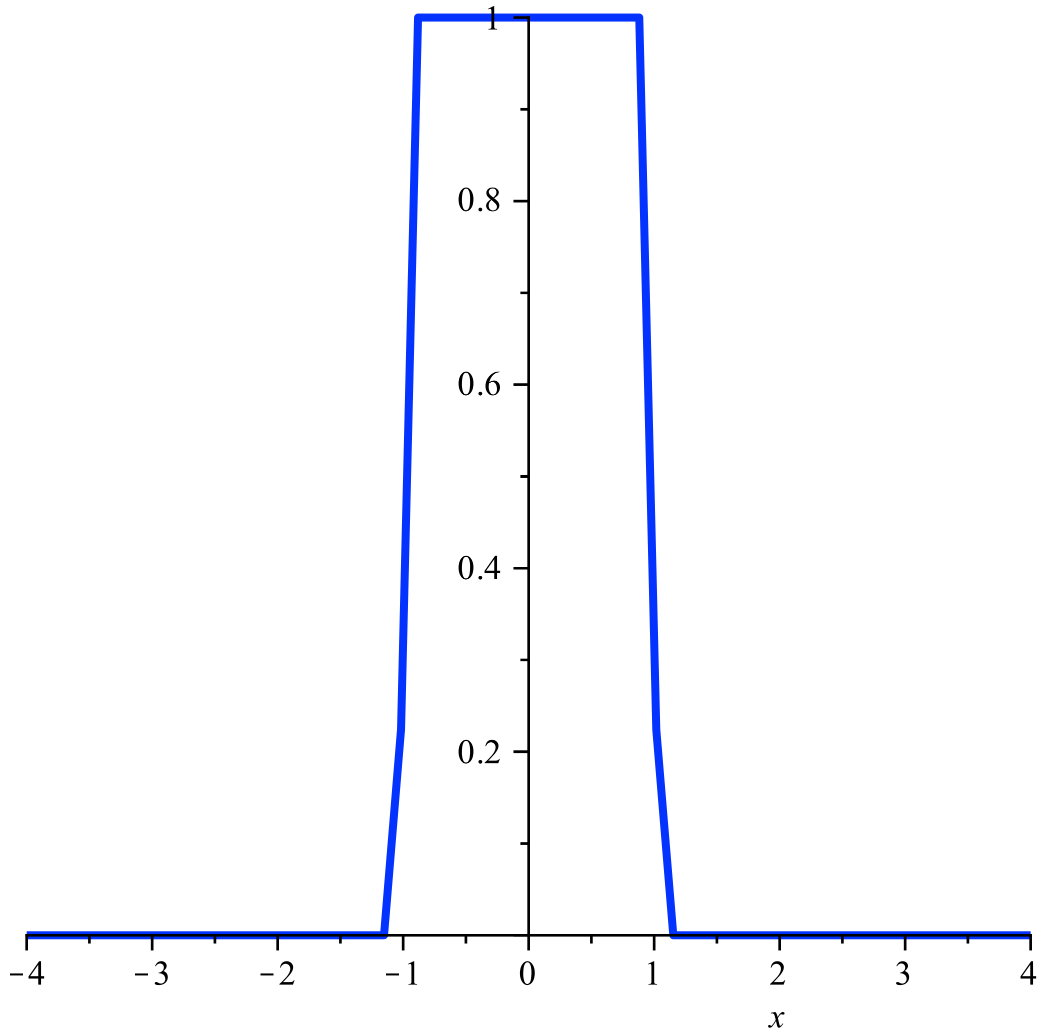


> $p4 := seq(plot(u2(x, 6\cdot i), x=-40..40, color=blue, thickness=2), i=10..20):$

> $display([p4], title=$ "Uniform profile (t=60..120)");



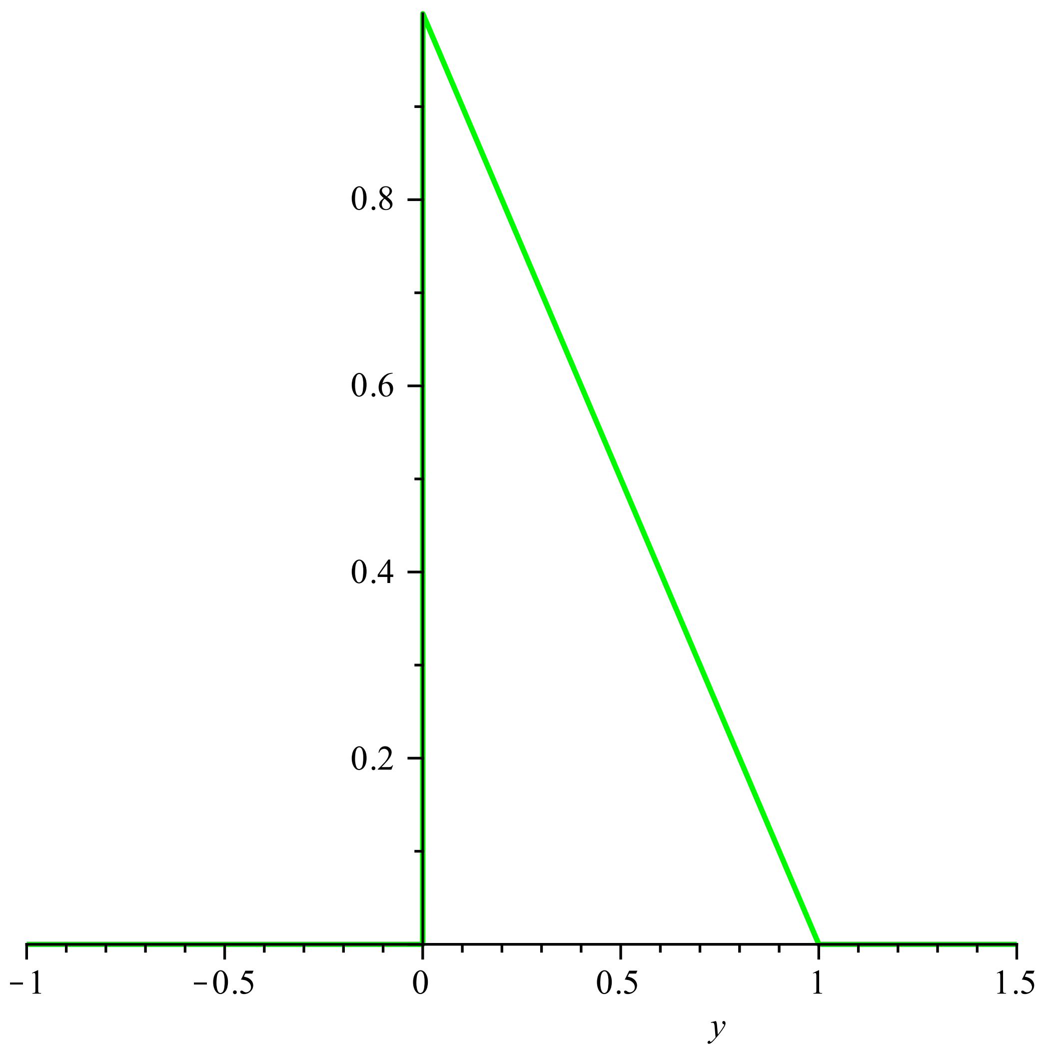
> *animate*(*u2*(*x*, *i*), *x* =−4 ..4, *i* = 0.001 ..2, *color* = *blue*, *thickness* = 3, *numpoints* = 60);



> $f3 := y \rightarrow (1 - y) \cdot (\text{Heaviside}(y) - \text{Heaviside}(y - 1));$

$$f3 := y \rightarrow (1 - y)\ (\text{Heaviside}(y) - \text{Heaviside}(y - 1)) \tag{20}$$

> $plot(f3(y), y = -1 ..1.5, thickness = 2, color = green)$



> $Su3 := simplify(value(subs(f=f3, Su)))$

$$Su3 := \frac{1}{2} \frac{1}{\sqrt{t\sim}\sqrt{\pi}} \left(-2\, c\sim\, t\sim\, \mathrm{e}^{-\frac{1}{4} \frac{x^2}{c\sim^2\, t\sim}} - \sqrt{t\sim}\, x \sqrt{\pi}\, \operatorname{erf}\left(\frac{1}{2} \frac{x}{c\sim\sqrt{t\sim}}\right) + 2\, c\sim\, t\sim\, \mathrm{e}^{-\frac{1}{4} \frac{(-1+x)^2}{c\sim^2\, t\sim}} + \sqrt{t\sim}\, x \sqrt{\pi}\, \operatorname{erf}\left(\frac{1}{2} \frac{-1+x}{c\sim\sqrt{t\sim}}\right) + \sqrt{\pi}\sqrt{t\sim}\, \operatorname{erf}\left(\frac{1}{2} \frac{x}{c\sim\sqrt{t\sim}}\right) - \sqrt{\pi}\sqrt{t\sim}\, \operatorname{erf}\left(\frac{1}{2} \frac{-1+x}{c\sim\sqrt{t\sim}}\right) \right) \quad \textbf{(21)}$$

>