

PDE and Boundary-Value Problems (Winter Term 2014/2015) Assignment H1 - Homework

Problem 1.1 (Classification - 4 Points)

Classify the following equations:

- (a) $u_t = u_{xx} + 2u_x + u$
- (b) $u_t = u_{xx} + e^{-t}$
- (c) $u_{xx} + 3u_{xy} + u_{yy} = sinx$
- (d) $u_{tt} = u u_{xxxx} + e^{-t}$

Problem 1.2 (Transformation into normal form - 12 Points)

Transform into normal form the following equations

(a)
$$u_{xx} - 2xu_{xy} - \frac{1}{x}u_x = 0, \qquad x > 0,$$

(b)
$$u_{xx} + 2u_{xy} + x^2 u_x = e^{-x^2/2}$$

Problem 1.3 (Definition of the type - 9 Points)

Define the type (elliptic, parabolic, etc.) of the following equations

- (i) $xu_{xx} + 2xu_{xy} + (x-1)u_{yy} = 0$,
- (ii) $u_{xy} 2u_{xz} + u_{yz} + u_x + \frac{1}{2}u_y = 0,$
- (iii) $u_{xx} + 2u_{xy} + 2u_{xz} + u_{yy} + 2u_{yz} + u_{zz} u = 0.$

Problem 1.4 (Solving PDE - 5 Points)

Can you find all functions u(x, y) that satisfy to the equation

$$\frac{\partial^2 u(x,y)}{\partial x \partial y} = 0?$$

How many are there?

Deadline for submission: Monday, November 3, (Room 003, Bilding E1 3)