Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
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UNIVERSITÄT DES SAARLANDES

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PDE and Boundary-Value Problems (Winter Term 2014/2015)
Assignment H3 - Homework

Problem 3.1 (Transformation of IBVP - 6 Points)

Transform

$$\text{PDE:}\quad u_t = u_{xx}, \qquad 0 < x < 1, \quad 0 < t < \infty$$

$$\text{BCs:}\quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 1 \end{cases}, \qquad 0 < t < \infty$$

$$\text{IC:}\quad u(x,0) = x^2, \qquad 0 \leqslant x \leqslant 1$$

to zero BCs and solve the new problem. What is the steady-state solution?

Problem 3.2 (Solving the IBVP - 8 Points)

Solve the problem

$$\text{PDE:}\quad u_t(x,y,t) = u_{xx}(x,y,t) + u_{yy}(x,y,t), \qquad 0 < x < 1, \quad 0 < y < 1, \quad 0 < t < \infty$$

$$\text{BCs:}\quad \begin{cases} u_x(0,y,t) = 0 \\ u_x(1,y,t) = -u(1,y,t) \\ u(x,0,t) = 0 \\ u(x,1,t) = 0 \end{cases} \qquad 0 < t < \infty$$

$$\text{IC:}\quad u(x,y,0) = \left(1 - \tfrac{x^3}{3}\right) y(1-y), \qquad 0 < x < 1, \quad 0 < y < 1$$

by using the method of separation of variables.

Problem 3.3 (Laplace Transform - 6 Points)

Solve the problem

$$\begin{aligned} \text{PDE:} \quad & u_t = u_{xx}, && 0 < x < \infty, \quad 0 < t < \infty \\ \text{BC:} \quad & u(0,t) = \sin(t), && 0 < t < \infty \\ \text{IC:} \quad & u(x,0) = 0, && 0 \leqslant x < t\infty \end{aligned}$$

by means of the Laplace transform (transform t). What is the physical interpretation of this problem?

Deadline for submission: Monday, Deember 1, 12 pm