Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
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PDE and Boundary-Value Problems (Winter Term 2014/2015)
Assignment H4 - Homework

Problem 4.1 (Fourier Transform and Maple - 3x4=12 Points)

Using Maple solve the problem

$$\text{PDE:} \quad u_t = u_{xx}, \qquad -\infty < x < \infty, \quad 0 < t < \infty$$

$$\text{IC:} \quad u(x,0) = \varphi(x), \qquad \infty < x < \infty$$

by means of the Fourier transform. Compare the solutions for the following values of the initial data:

(a) $\varphi(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases},$

(b) $\varphi(x) = \begin{cases} \cos(x), & |x| < \pi/2 \\ 0, & |x| > \pi/2 \end{cases},$

(c) $\varphi(x) = xe^{-2|x|}$.

Problem 4.2 (Laplace Transform and Maple - 8 Points)

We seek the temperature distribution $u(x,t)$ in a thin rod over the interval $0 < x < \infty$ whose lateral surface is insulated. The left end of the rod is constarined at a varying temperature $q(t) = \dfrac{1}{\sqrt{t}}$, and the rod has an initial temperature distribution $u(x,0) = 0$. There is no heat source in the system and the termal diffusivity of the rod is $k = 1/50$.

Using Maple solve the problem by means of the Laplace transform (transform t). Show the animation of the spatial-time temperature distribution $u(x,t)$ in the rod for $0 \leqslant t \leqslant 5$.

(*Hinweis:* use the Maple commands **laplace** and **invlaplace** for the Laplace transform and the inverse Laplace transform, respectively.)

Problem 4.3 (Solving IBVP - 6 Points)

Suppose we have a metal rod (laterally insulated) and we supply an *initial impulse of heat* at the right-hand side (the left-hand side is fixed at zero). Suppose the initial temperature of the rod is zero (some reference temperature) and the temperature at the midpoint $x = 0.5$ is measured at various values of time, so that we have the following table:

Values of Time	**Midpoint Temperature**
t_1	w_1
$t_2 = 2t_1$	w_2
$t_3 = 3t_1$	w_3
$\vdots$	$\vdots$
$t_n = nt_1$	w_n

Using this data, how could we approximate the temperature responce at the points $u(0.5, t_n)$ due to the BCs

(a) $u(1, t) = \sin(t)$

(b) $u(x, t) = f(t)$ (arbitrary $f(t)$)?

Problem 4.4 (Duhamel's Principle - 4 Points)

Using Duhamel's principle, what is the solution of the IBVP

$$\text{PDE:} \quad u_t = \alpha^2 u_{xx}, \qquad 0 < x < 1, \quad 0 < t < \infty$$

$$\text{BCs:} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = \sin(t) \end{cases}, \qquad 0 < t < \infty$$

$$\text{IC:} \quad u(x,0) = 0, \qquad 0 \leqslant x \leqslant 1$$

Deadline for submission: Monday, December 15, 12pm