UNIVERSITÄT DES SAARLANDES

PDE and Boundary-Value Problems (Winter Term 2014/2015)
Assignment H5 - Homework

Problem 5.1 (Solving IBVP - 6 Points)

Solve the semi-infinite string problem:

$$\text{PDE:}\quad u_{tt} = u_{xx}, \qquad\qquad 0 < x < \infty, \quad 0 < t < \infty$$

$$\text{BC:}\quad u(0,t) = 0, \qquad\qquad 0 < t < \infty$$

$$\text{ICs:}\quad \begin{cases} u(x,0) = xe^{-x^2} \\ u_t(x,0) = 0 \end{cases}, \qquad 0 < x < \infty$$

Problem 5.2 (Solving the IBVP - 6 Points)

Find the solution to the following vibrating-string problem

$$\text{PDE:}\quad u_{tt} = \alpha^2 u_{xx}, \qquad\qquad\qquad 0 < x < L, \quad 0 < t < \infty$$

$$\text{BCs:}\quad \begin{cases} u(0,t) = 0 \\ u(L,t) = 0 \end{cases}, \qquad\qquad 0 < t < \infty$$

$$\text{ICs:}\quad \begin{cases} u(x,0) = \sin\left(\pi x/L\right) + 0,5\sin\left(3\pi x/L\right) \\ u_t(x,0) = 0 \end{cases}, \qquad 0 \leqslant x \leqslant L$$

Problem 5.3 (The Vibrating Beam - 7 Points)

What is the solution to the simply supported (at both ends) beam problem with ICs

$$\begin{aligned} u(x,0) &= \sin\left(\pi x\right) \\ u_t(x,0) &= \sin\left(\pi x\right) \end{aligned} \qquad 0 \leqslant x \leqslant 1$$

Deadline for submission: Monday, January 12, 12pm