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Fachrichtung Mathematik
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Algebraic Geometry Summer Term 2018

Exercise Sheet 12. Hand in by Friday, July 13.

Exercise 1. Let $p_0, \dots, p_{n+1} \in \mathbb{P}^n$ be $n+2$ points, such that no $n+1$ of these points lie on a hyperplane. Prove that there exist a unique automorphism $A \in PGL(n+1, \overline{K})$, which maps these points to the points $(1:0:\dots:0), (0:1:0:\dots:0), \dots, ((0:\dots:0:1)$ and $(1:1:\dots:1)$.

Exercise 2. Compute a rational paramerization of the plane curve defined by

$$\begin{aligned} f =& x^5 + 10x^4y + 20x^3y^2 + 130x^2y^3 - 20xy^4 + 20y^5 \\ &- 2x^4 - 40x^3y - 150x^2y^2 - 90xy^3 - 40y^4 \\ &+ x^3 + 30x^2y + 110xy^2 + 20y^3 \in \mathbb{Q}[x,y] \end{aligned}$$

with the help of the Macaulay2 or some other Computer algebra system.

Exercise 3. Consider the plane curves $A_n = V(y^2 - x^{n+1})$ and $E_8 = V(y^3 - x^5)$. Compute a resolution the singularity at the origin via successive blow-ups.

Exercise 4.
Consider the projective closure of the curve X defined by

$$y^2 = x(x^2-1)(x^2-4)$$

in $\mathbb{A}^1 \times \mathbb{A}^1 \subset \mathbb{P}^1 \times \mathbb{P}^1$. What is the genus of this curve? How does the underlying 2-dimensional real manifold $X(\mathbb{C})$ looks like?