UNIVERSITÄT DES SAARLANDES
Fachrichtung Mathematik
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Computer Algebra   Summer Term 2019

Exercise Sheet 1. Hand in by Tuesday, April 23.

Exercise 1
Prove: $\mathbb{Z}[i] = \{a + ib \in \mathbb{C} \mid a, b \in \mathbb{Z}\}$ together with

$$d : \mathbb{Z}[i] \to \mathbb{N}, a + ib \mapsto a^2 + b^2$$

is an euclidean domain.

Exercise 2
Prove: Every finite integral domain is a field.

Exercise 3
Consider $R = \mathcal{C}[0,1] = \{f : [0,1] \to \mathbb{R} \mid f \text{ is continuous}\}$ and the subset

$$J = \{f \in R \mid f(\frac{1}{n}) = 0 \text{ for all but finitely many } n \in \mathbb{N}\}.$$

Prove: $J \subset R$ is an ideal, which is not finitely generated.

Exercise 4
Prove Noether's isomorphism theorems:

(1) Let $P \subset N \subset M$ be submodules of a module M. Then N/P is a submodule of M/P and

$$(M/P)/(N/P) \cong M/N.$$

(2) Let N, M be submodules of a module P. Then $N \cap M$ and $N + M = \{n + m \mid n \in N, m \in M\}$ are submodules of P and

$$M/(M \cap N) \cong (M + N)/N.$$