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Fachrichtung Mathematik
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Computer Algebra   Summer Term 2019

Exercise Sheet 10. Hand in by Tuesday, June 25.

Exercise 1. Let $A \subset \mathbb{A}^n$ be an algebraic set defined by an ideal $I \subset K[x_1, \dots, x_n]$. Let $\overline{A} \subset \mathbb{P}^n$ denote the closure of $A \subset \mathbb{A}^n \cong U_0 \subset \mathbb{P}^n$ in the Zariki topology. Prove that $\overline{A}$ is defined by the ideal

$$I^h = \langle \{ f^h \mid f \in I \} \rangle \subset K[x_0, \dots, x_n].$$

Exercise 2. Let $I \subset K[x_1, \dots, x_n]$ be an ideal and $>$ a monomial order which refines the total degree, i.e.

$$\deg m_1 > \deg m_2 \implies m_1 > m_2 \text{ for monomials } m_1, m_2.$$

Let $f_1, \dots, f_r$ be a Gröbner basis of I with respect to $>$. Prove: $I^h = \langle f_1^h, \dots, f_r^h \rangle$.

Exercise 3. Draw pictures of the real points of the curve defined by $x^2 + (y-1)^2 = 1$ in all three charts of $\mathbb{P}^2(\mathbb{R})$.

Exercise 4.
Suppose the quadrangle with corners $(0,0), (1,-1), (-1,-5)$ and $(-2,-2) \in \mathbb{R}^2$ is the perspective drawing of a tiling of the plane into squares. Draw three more tiles!