UNIVERSITÄT DES SAARLANDES
Fachrichtung Mathematik
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Computer Algebra   Summer Term 2019

Exercise Sheet 4. Hand in by Tuesday, May 14.

Exercise 1

$f \in K[x_1, \ldots, x_n]$ be a non-zero polynomial. Prove: A change of coordinates

$$\varphi : K[x_1, \ldots, x_n] \to K[x_1, \ldots, x_n]$$

defined by

$$x_1 \mapsto x_1, x_i \mapsto x_i + x_1^{r^i}$$

for $r \in \mathbb{N}$ a sufficiently large transforms λf for a suitable scalar $\lambda \in K \setminus \{0\}$ into an x_1-monic polynomial.

Exercise 2

Let $\mathbb{F}_p = \mathbb{Z}/p$ be a finite field. Show that there exists a polynomial $f \in \mathbb{F}[x, y]$ such that no linear change of coordinates with coefficients in $\mathbb{F}$ transform f into an up to scalar x-monic polynomial.

Exercise 3

Let $A = C_1 \cup \ldots \cup C_r \subset \mathbb{A}^n$ be an algebraic set decomposed into its irreducible components. Prove:

$$\dim A = \max\{\dim C_i \mid i = 1, \ldots r\}$$

Exercise 4

Check that the polynomials

$$f_1 = x^3 - xz,\ f_2 = yx^2 - yz \in K[x, y, z]$$

form a lexicographic Gröbner basis. Conclude that $A = V(f_1, f_2)$ projects onto the yz-plane. Determine the points of the yz-plane with 1,2, and 3 preimage points, respectively.