UNIVERSITÄT DES SAARLANDES
Fachrichtung Mathematik
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Computer Algebra Summer Term 2019

Exercise Sheet 9. Hand in by Tuesday, June 18.

Exercise 1. Let

$$f = x^3 - 292x^2 - 2170221x + 6656000 \in \mathbb{Z}[x].$$

Find 13-adic linear factors $x - a_i$ such that the remainder of f divided by $x - a_i$ is $\equiv 0 \mod 13^{2^i}$ for $i = 0, 1, 2$ starting with $a_0 = 0$.

Exercise 2. Compute the coefficients of the Swinnerton-Dyer polynomial

$$f = (x + \sqrt{-1} + \sqrt{2})(x + \sqrt{-1} - \sqrt{2})(x - \sqrt{-1} + \sqrt{2})(x - \sqrt{-1} - \sqrt{2}) \in \mathbb{Z}[x]$$

and its factorization modulo $p = 2, 3, 5$.

Prove that f is irreducible.

Exercise 3. Prove Eisenstein's theorem: If $f \in \mathbb{Z}[x]$ and p a prime number such that $p \nmid lc(f)$, p divides all other coefficients of f, and $p^2 \nmid f(0)$, then f is irreducible in $\mathbb{Q}[x]$.

Conclude that for any $n \in \mathbb{N}$ the polynomial $x^n - p$ is irreducible in $\mathbb{Q}[x]$.

Exercise 4. Let K be a field. Prove that $K[[t]]$ is a PID.

Why are the rings $K[[t]][x]$, $K[x][[t]]$ and $K[[t, x]]$ not equal?