Fakultät MI, Fachrichtung Mathematik
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UNIVERSITÄT DES SAARLANDES

Mathematics for computer science 1

Winterterm 2019/20

Hand in your solution sheet in the mailboxes (next to Zeichensaal U.39, building E2 5) by Jan. 22 **before the lecture**.

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 11 15. January 2019

Exercise 1 (Concave functions)**.** Let I be an interval, let $f : I \to \mathbb{R}$ be a concave function and let $x_1, \ldots, x_n \in I$.

(a) Show that

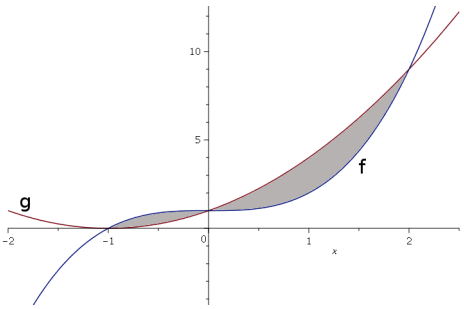
$$\sum_{i=1}^{n} \frac{f(x_i)}{n} \leq f\left(\sum_{i=1}^{n} \frac{x_i}{n}\right).$$

(b) Conclude the inequality between the geometric and arithmetic average for $x_i > 0$

$$\sqrt[n]{x_1 \cdot \ldots \cdot x_n} \leq \sum_{i=1}^{n} \frac{x_i}{n}.$$

(Hint: choose $f = \ln$.)

Exercise 2 (Area)**.** Let $f, g : \mathbb{R} \to \mathbb{R}$ be functions defined by $f(x) = x^3 + 1$ and $g(x) = (x+1)^2$. Determine the area between the two graphs of f and g, that is, the grey area in the picture below.



Exercise 3 (Limits)**.** (a) Show:

$$\lim_{x\to 0} \frac{2\cos x + e^x + e^{-x} - 4}{x^4} = \frac{1}{6}, \quad \lim_{x\to 0} \frac{\sqrt{\cos ax} - \sqrt{\cos bx}}{x^2} = \frac{b^2 - a^2}{4} \quad \text{for } a, b \in \mathbb{R}.$$

Compute the following limits, if existent:

(b) $\lim_{x \searrow 0} \frac{\ln x}{\cot x}$,

(c) $\lim_{x \to \frac{\pi}{2}} \frac{\tan(3x)}{\tan(x)}$,

(d) $\lim_{x \searrow 1} \big(\ln(x) \cdot \ln(1-x)\big)$.

Exercise 4 (The Euler number)**.** Show that $\lim_{n\to\infty}\big(n \ln(1 + \frac{1}{n})\big) = 1$ and conclude that

$$\lim_{n\to\infty} \big(1 + \frac{1}{n}\big)^n = e.$$