Mathematics for computer science 1

Winterterm 2019/20

Hand in your solution sheet in the mailboxes (next to Zeichensaal U.39, building E2 5) by Oct. 30 **before the lecture**.
The tutorials will start on Monday, Oct. 28, with an exercise sheet 0 (solved and corrected in the tutorial).

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 1 23. October 2019

Exercise 1 (Tower of Hanoi). The game *The Towers of Hanoi* consists of three rods and a number $n \in \mathbb{N}$ of disks of different sizes, which can slide onto any rod. The puzzle starts with the disks in a neat stack in ascending order of size on one rod (the smallest at the top, as seen in the picture). The goal of the game is to move the entire stack to another rod such that it is in the same stacking order again. Each move consists of taking only the upper disk from one of the stacks and placing it on top of another stack or on an empty rod. No larger disk may be placed on top of a smaller disk.
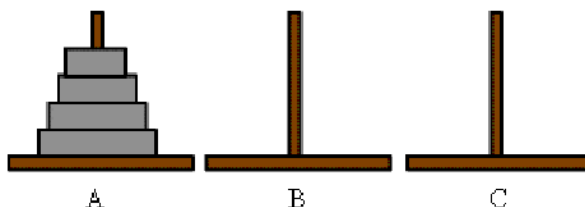
Provide an algorithm (only on paper) that solves this problem and prove the correctness of your algorithm. Give a formula for the number of moves required and prove it by induction.

Exercise 2 (Induction). Prove by induction:

(a) $\sum_{k=0}^{n} 3^k = \frac{3^{n+1}-1}{2}$,

(b) $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

Exercise 3 (Four witnesses). An inspector has heard four witnesses to a crime. From the interrogations he has drawn the following conclusions:

- If the **B**utler is telling the truth, then the **C**ook is too.
- The Cook and **G**ardener cannot both tell the truth.
- The Gardener and **J**anitor do not both lie.
- If the Janitor is telling the truth, then the Cook is lying.

(a) Model the inspector's information as logical formulas. Use the variables B, C, G and J.

(b) For which witnesses can the inspector be sure that they are lying? For which can he be sure that they are telling the truth? Explain how you come to your conclusion!

Exercise 4 (Two investment bankers). A man is consulting two investment bankers, say A and B. He wants to find out whether he would rather invest his inheritance in investment option 1 or 2. Unfortunately, the bank's free advice allows one to ask only one "Yes or No" question to one of the two consultants. A friend had previously informed him that one of them always tells the truth and that the other one always lies. Unfortunately, the friend has forgotten who is who. Formulate a question to find out what the good and bad investment option are.