Fakultät MI, Fachrichtung Mathematik
Prof. Dr. Frank-Olaf Schreyer
Dr. Michael Hoff

Mathematics for computer science 1

Winterterm 2019/20

Hand in your solution sheet in the mailboxes (next to Zeichensaal U.39, building E2 5) by Dec. 18 **before the lecture**.

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 8 11. December 2019

Exercise 1 (Continuity)**.** Determine all points in $\mathbb{R}$ where the following function is continuous:

$$f(x) = \begin{cases} -x+1, & x \leq -1, \\ x^2+5x+7, & -1 < x \leq 0, \\ x+7, & x > 0. \end{cases}$$

Exercise 2. Let $x_0 \in \mathbb{R}$ and let $f : \mathbb{R} \to \mathbb{R}$ be a function.

(i) $\exists \delta > 0 \ \forall \varepsilon : \ \forall x$ with $|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$,

(ii) $\forall \varepsilon > 0 \ \exists \delta : \ \forall x$ with $|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$,

(iii) $\exists \varepsilon \ \forall \delta > 0 : \ \forall x$ with $|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$,

(iv) $\forall \varepsilon > 0 \ \forall \delta : \ \forall x$ with $|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$,

(iv) $\forall \varepsilon > 0 \ \forall x \ \exists \delta : \ |f(x) - f(x_0)| < \varepsilon \Rightarrow |x - x_0| < \delta$.

(a) Reformulate the five statements colloquially.

(b) Provide all implications between the following five statements. Give examples of functions which show that there are no further implications.

Exercise 3 (Addition theorem for sin and cos)**.**

(a) Show that for all $n \in \mathbb{N}$ there exist polynomials $p_n(x, y)$ and $q_n(x, y)$ in two variables x, y with real coefficients such that

$$\sin(nt) = p_n(\sin(t), \cos(t)) \quad \text{and} \quad \cos(nt) = q_n(\sin(t), \cos(t))$$

for all $t \in \mathbb{R}$.

(b) Compute $p_n(x, y)$ and $q_n(x, y)$ for $n = 2, 3, 4$.

Exercise 4. A cord is stretched between two walls in a room. Now the cord is released on both sides and somehow thrown into the middle of the room. Show that there is a point on the cord with the same distance to the walls as before.