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Computer Algebra and Gröbner Bases

Winterterm 2020/21

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 2 19. November 2020

Exercise 1. Let $I \subset K[x_1,\dots,x_n]$ be an ideal. Prove

$$V(I) \subset \mathbb{A}^n \text{ is finite } \iff K[x_1,\dots,x_n]/I \text{ is a finite dimensional } K\text{-vector space.}$$

Exercise 2. Consider the ideal $I = (xy(x+y)+1) \subset \mathbb{F}_2[x,y]$. Determine coordinates in which I satisfies the extra hypothesis of the projection theorem. Show that the extra hypothesis cannot be achieved by means of a linear change of coordinates.

Exercise 3. A binomial $f \in K[x_1,\dots,x_n]$ is a polynomial with has exactly two terms

$$f = ax^\alpha - bx^\beta.$$

A binomial ideal is an ideal generated by binomials and monomials. Prove:
Binomial ideals have a Gröbner basis consisting of binomials and monomials.

Exercise 4. (Key property of $>_{\text{lex}}$.)

(1) Let $f \in K[x_1,\dots,x_n]$ and $1 \le j \le n-1$.

$$\mathrm{Lt}_{\text{lex}}(f) \in K[x_{j+1},\dots,x_n] \iff f \in K[x_{j+1},\dots,x_n]$$

(2) Let $f_1,\dots,f_r$ be a Gröbner basis of $I \subset K[x_1,\dots,x_n]$ with respect to $>_{\text{lex}}$. Then

$$\{f_s \mid \mathrm{Lt}_{\text{lex}}(f_s) \in K[x_{j+1},\dots,x_n]\}$$

is a Gröbner basis of $I_j = I \cap K[x_{j+1},\dots,x_n]$.