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Reflexivity for subnormal systems with dominating spectrum in product domains

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Abstract. The question whether every subnormal tuple $S = (S_1, \ldots, S_n) \in L(H)^n$ on a complex Hilbert space is reflexive is one of the major open problems in multivariable invariant subspace theory. Positive answers have been given for subnormal tuples with rich spectrum in the unit polydisc or the unit ball. The ball case has been extended by Didas [6] to strictly pseudoconvex domains. In the present note we extend the polydisc case by showing that every subnormal tuple with pure components and rich spectrum in a bounded polydomain $U = U_1 \times \ldots \times U_n \subset \mathbb{C}^n$ is reflexive.

0 Introduction

Let H be a complex Hilbert space. For an arbitrary set $\mathcal{S} \subset L(H)$ of bounded linear operators on H, we denote by $\mathrm{Lat}(\mathcal{S})$ the lattice of all closed linear subspaces of H that are invariant under every operator $S \in \mathcal{S}$. The set

$$\mathrm{AlgLat}(\mathcal{S}) = \{\mathrm{C} \in \mathrm{L(H)}; \mathrm{Lat(C)} \subset \mathrm{Lat}(\mathcal{S})\}$$

is a unital subalgebra of $L(H)$ which contains $\mathcal{S}$ and is closed in the weak operator topology (WOT). The family $\mathcal{S}$ is called reflexive if

$$\mathrm{AlgLat}(\mathcal{S}) = \mathcal{W}_{\mathcal{S}},$$

where $\mathcal{W}_{\mathcal{S}}$ is the smallest WOT-closed subalgebra of $L(H)$ containing $\mathcal{S}$.

The notion of reflexivity was introduced by Sarason [14] who proved in 1966 that analytic Toeplitz operators on the Hardy space $H^2(\mathbb{D})$ and commuting families of normal operators are reflexive. In 1971 it was shown by Deddens [5] that every isometry on a Hilbert space is reflexive. All the previous single variable results were generalized in a paper of Olin and Thomson [13] from 1980 in which the Scott Brown technique was used to show that every subnormal operator on a Hilbert space is reflexive. The question whether every subnormal system $S = (S_1, \ldots, S_n) \in L(H)^n$, that is, every n-tuple $S \in L(H)^n$ that extends to a system $N = (N_1, \ldots, N_n) \in L(K)^n$ of

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commuting normal operators on a larger Hilbert space K, is reflexive, is one of the main open problems in this area of multivariable invariant subspace theory.

The result of Deddens on the reflexivity of single isometries was extended to families of commuting isometries by Li and McCarthy [12] and Bercovici [2], and to spherical isometries by Didas [7]. Both, commuting tuples of isometries and spherical isometries, are examples of subnormal systems.

To indicate some of the known reflexivity results for general subnormal systems, let us recall that a given compact set $K \subset \mathbb{C}^n$ is said to be dominating in an open set $U \subset \mathbb{C}^n$ if $K \subset \overline{U}$ and if the supremum-norms of any bounded analytic function $f \in H^\infty(U)$ on U and on $U \cap K$ satisfy

$$\|f\|_U = \|f\|_{K \cap U}.$$

One way to prove the Olin and Thomson result on the reflexivity of single subnormal operators is to show that the spectrum of a pure subnormal operator $S \in L(H)$ is dominating in a bounded open set U in $\mathbb{C}$ with simply connected components such that S possesses an isometric weak*-SOT continuous H^∞-functional calculus on U. Then a canonical decomposition of the space H, together with the Riemann mapping theorem, allows the reduction of the general problem to the case of a subnormal contraction $S \in L(H)$ of type $C_{\cdot 0}$ with isometric H^∞-functional calculus $\Phi : H^\infty(\mathbb{D}) \to L(H)$ over the open unit disc.

In the multivariable case only parts of this programme have been realized so far. Let $S = (S_1, \ldots, S_n) \in L(H)^n$ be a subnormal tuple of pure subnormal operators S_ν $(1 \leq \nu \leq n)$. It was shown in [8] and [9] that S is reflexive if its Taylor spectrum $\sigma(S)$ is dominating in the open Euclidean unit ball $\mathbb{B}_n \subset \mathbb{C}^n$ or in the open unit polydisc $\mathbb{D}^n \subset \mathbb{C}^n$. It was observed by Didas [7] that the Euclidean unit ball can be replaced by any strictly pseudoconvex bounded open set. In general, the question to decide for which classes of open sets U in $\mathbb{C}^n$ a result of the above type holds, seems to be difficult. It is the purpose of the present note to show that the answer is positive for subnormal tuples S with pure components S_ν and dominating spectrum in a bounded open polydomain $U = U_1 \times \ldots \times U_n$ in $\mathbb{C}^n$.

1 Preliminaries

Let H be a complex Hilbert space, and let $L(H)$ be the Banach algebra of all continuous linear operators on H. We regard $L(H)$ as the norm-dual of the space $\mathcal{C}^1(H)$ of all trace-class operators on H. Let $T = (T_1, \ldots, T_n) \in L(H)^n$

be a commuting tuple. Then the smallest weak* closed unital subalgebra $\mathcal{A}_T$ of $L(H)$ containing $T_1.\ldots,T_n$ is isometrically isomorphic to the norm-dual of the quotient space $Q(T) = \mathcal{C}^1(H)/^{\perp}\mathcal{A}_T$. In this way $\mathcal{A}_T$ becomes a dual algebra, that is, $\mathcal{A}_T$ is the norm-dual of a suitable Banach space such that the multiplication in $\mathcal{A}_T$ is separately weak* continuous. If A and B are dual algebras, then a dual algebra isomorphism $\varphi : A \to B$ is by definition an algebra homomorphism between A and B that is an isometric isomorphism and a weak* homeomorphism.

For $x, y \in H$, let us denote by $[x \otimes y] \in Q(T)$ the equivalence class of the rank-one operator $H \to H, \xi \mapsto \langle \xi, y\rangle x$. For a given real number $r \geq 1$, the dual algebra $\mathcal{A}_T$ is said to have property $(\mathbb{A}_1(r))$ if, for every $s > r$ and every element $[L] \in Q(T)$, there are vectors $x, y \in H$ with $[L] = [x \otimes y]$ and

$$\|x\|, \|y\| \;\leq\; (s\|[L]\|)^{1/2}.$$

For a bounded open set $U \subset \mathbb{C}^n$, the Banach algebra $H^\infty(U)$ of all bounded analytic functions on U is regarded as the norm-dual of the quotient $Q = L^1(U)/^{\perp}H^\infty(U)$. Here we use that $H^\infty(U)$ is a weak* closed subspace of $L^\infty(U)$ relative to the duality $\langle L^1(U), L^\infty(U)\rangle$ (formed with respect to the $(2n)$-dimensional Lebesbue measure). A sequence (f_k) in $H^\infty(U)$ is weak* convergent to zero if and only if (f_k) is norm-bounded and converges to zero pointwise on U. A given compact set $K \subset \mathbb{C}^n$ is said to be dominating in U if $K \subset \overline{U}$ and

$$\|f\|_U = \|f\|_{K\cap U}$$

holds for every function $f \in H^\infty(U)$, where the norms are the supremum-norms on U and $K \cap U$. If there is a bounded open set $U \subset \mathbb{C}^n$ such that K is dominating in U, then the union of all these open sets U is the largest bounded open set in which K is dominating.

Let $\Phi : H^\infty(U) \to L(H)$ be a continuous algebra homomorphism over a bounded open set $U \subset \mathbb{C}^n$. Then

$$\tilde{\Phi} : H^\infty(U^*) \to L(H), \quad f \mapsto \Phi(\tilde{f})^* \quad (\tilde{f}(z) = \overline{f(\bar{z})})$$

defines a continuous algebra homomorphism over the bounded open set $U^* = \{\bar{z};\ z \in U\}$. The algebra homomorphism Φ is by definition of type $C_{0\cdot}$ if $(\Phi(f_k)) \xrightarrow{k} 0$ in the strong operator topology for every weak* zero sequence (f_k) in $H^\infty(U)$. We say that Φ is of type $C_{\cdot 0}$ if $\tilde{\Phi}$ is of type $C_{0\cdot}$.

For a commuting tuple $T = (T_1, \ldots, T_n) \in L(H)^n$, we denote by $\sigma(T)$ its spectrum in the sense of J. L. Taylor. For the definition and basic properties of this notion of joint spectrum, we refer the reader to [10]. As usual we call a commuting tuple T pure if there is no non-zero reducing subspace M for T such that $T|M$ is a commuting tuple of normal operators.

2 Main result

Let $S = (S_1, \ldots, S_n) \in L(H)^n$ be a subnormal tuple such that all components S_ν $(1 \le \nu \le n)$ are pure subnormal operators on H. Then each component S_ν of S possesses an isometric weak* continuous functional calculus

$$\Phi_\nu : H^\infty(U_\nu) \to L(H)$$

of type $C_{\cdot 0}$ over the largest bounded open set $U_\nu \subset \mathbb{C}$ in which the spectrum of S_ν is dominating (Theorem 19.1.4 in [4]). Let us write each of the sets U_ν as the disjoint union of its connected components

$$U_\nu = \bigcup (C_{\nu k};\ 0 \le k < N_\nu) \qquad (\nu = 1, \ldots, n)$$

with suitable $N_\nu \in \{0, 1, 2, \ldots\} \cup \{\infty\}$.

Denote by $e_{\nu k} \in H^\infty(U_\nu)$ $(\nu = 1, \ldots, n,\ 0 \le k < N_\nu)$ the characteristic functions of the sets $C_{\nu k}$. Since the maps Φ_ν are contractive and since the polynomials are weak* dense in $H^\infty(U_\nu)$ (Theorem 19.1.2 in [4]), the operators $P_{\nu k} = \Phi_\nu(e_{\nu k}) \in L(H)$ are mutually commuting orthogonal projections on H such that the range spaces $H_{\nu k} = P_{\nu k} H$ are reducing subspaces for $\Phi_1, \ldots, \Phi_n$. Furthermore, for each fixed $\nu = 1, \ldots, n$, the projections $(P_{\nu k})_{0 \le k < N_\nu}$ yield an orthogonal decomposition of H of the form

$$H = \bigoplus (H_{\nu k};\ 0 \le k < N_\nu).$$

Denote by I the set of all index tuples $i = (i_1, \ldots, i_n)$ consisting of nonnegative integers $0 \le i_\nu < N_\nu$ for $\nu = 1, \ldots, n$. Then the closed subspaces

$$H_i = \left(\prod_{\nu=1}^{n} P_{\nu, i_\nu} \right) H = \bigcap_{\nu=1}^{n} H_{\nu, i_\nu} \qquad (i \in I)$$

are reducing for $\Phi_1, \ldots, \Phi_n$ and yield an orthogonal decomposition

$$H = \bigoplus_{i \in I} H_i\,.$$

For a given function $f \in H^\infty(C_{\nu k})$, we denote by $\tilde{f} \in H^\infty(U_\nu)$ its trivial extension to the open set U_ν. Since the polynomials are weak* dense in $H^\infty(U_\nu)$ (Theorem 19.1.2 in [4]) and since the restriction mappings from $H^\infty(U_\nu)$ to $H^\infty(C_{\nu k})$ are weak* continuous, it follows that the polynomials are weak* dense in each of the spaces $H^\infty(C_{\nu k})$.

Let us fix an index tuple $i = (i_1, \ldots, i_n) \in I$. Then the mappings

$$\Phi_{\nu i} : H^\infty(C_{\nu, i_\nu}) \to L(H_i), \quad f \mapsto \Phi_\nu(\tilde{f})|H_i \qquad (\nu = 1, \ldots, n)$$

are contractive weak* continuous algebra homomorphisms of type $C_{\cdot 0}$ such that $\mathrm{Im}\Phi_{\nu i} \subset \mathfrak{A}_{(S_\nu|H_i)}$. Since the components of U_ν are simply connected domains in $\mathbb{C}$, the Riemann mapping theorem allows us to choose conformal mappings

$$\varphi_\nu : C_{\nu,i_\nu} \to \mathbb{D} \qquad (\nu = 1, \ldots, n)$$

onto the open unit disc $\mathbb{D}$ in $\mathbb{C}$. By composing the representations $\Phi_{\nu i}$ with the dual algebra isomorphisms

$$H^\infty(\mathbb{D}) \to H^\infty(C_{\nu,i_\nu}), \quad f \mapsto f \circ \varphi_\nu \quad (\nu = 1, \ldots, n),$$

we obtain contractive weak* continuous functional calculi

$$\Psi_{\nu i} : H^\infty(\mathbb{D}) \to L(H_i), \quad f \mapsto \Phi_{\nu i}(f \circ \varphi_\nu)$$

of type $C_{\cdot 0}$ for the operators $S_{\nu i} = \Phi_{\nu i}(\varphi_\nu) \in \mathfrak{A}_{(S_\nu|H_i)}$.

Since $S \in L(H)^n$ was supposed to be subnormal, every finite tuple of elements in $\mathfrak{A}_S$ is subnormal again (see the remarks following Theorem 1.8 in [8]). In particular, the tuple

$$S^{(i)} = (S_{1i}, \ldots, S_{ni}) \in L(H_i)^n$$

is subnormal and consists of $C_{\cdot 0}$-contractions. By a result of Apostol (Theorem 1.7 and Proposition 1.8 in [1]) the n-tuple $S^{(i)}$ possesses a contractive weak* continuous functional calculus

$$\Phi^{(i)} : H^\infty(\mathbb{D}^n) \to L(H_i)$$

which is again of type $C_{\cdot 0}$. Define $C_i = \prod_{\nu=1}^n C_{\nu,i_\nu} \subset \mathbb{C}^n$ and denote by $\rho_i : H^\infty(C_i) \to H^\infty(\mathbb{D}^n)$ the dual algebra isomorphism acting as

$$\rho_i(f)(z_1, \ldots, z_n) = f\big(\varphi_1^{-1}(z_1), \ldots, \varphi_n^{-1}(z_n)\big).$$

Then the composition

$$\Phi_i : H^\infty(C_i) \xrightarrow{\rho_i} H^\infty(\mathbb{D}^n) \xrightarrow{\Phi^{(i)}} L(H_i)$$

is a contractive weak* continuous functional calculus of type $C_{\cdot 0}$ for the subnormal n-tuple $S|H_i$. To check this, the reader should observe that

$$\Psi_{\nu i}(f) = \Phi^{(i)}(f \circ \pi_\nu) \qquad (\nu = 1, \ldots, n, f \in H^\infty(\mathbb{D})),$$

since both sides define weak* continuous representations of $H^\infty(\mathbb{D})$ that agree on polynomials. Here $\pi_\nu : \mathbb{D}^n \to \mathbb{D}$ denotes the projection onto the ν-th coordinate.

Define $U = U_1 \times \ldots \times U_n$. It is elementary to check that

$$\Phi : H^\infty(U) \to L(H), \quad \Phi(f) = \bigoplus_{i \in I} \Phi_i(f|C_i)$$

defines a contractive weak* continuous functional calculus of type $C_{\cdot 0}$ for S. Thus we have proved the following result.

2.1 Theorem. *Let $S = (S_1, \dots, S_n) \in L(H)^n$ be a subnormal tuple of pure subnormal operators S_ν. For $\nu = 1, \dots, n$, let $U_\nu \subset \mathbb{C}$ be the largest bounded open set in $\mathbb{C}$ such that $\sigma(S_\nu)$ is dominating in U_ν. Then S possesses a contractive weak* continuous functional calculus*

$$\Phi : H^\infty(U) \to L(H)$$

of type $C_{\cdot 0}$ over the product set $U = U_1 \times \ldots \times U_n$.

Let us suppose in addition that the Taylor spectrum $\sigma(S)$ of S is dominating in U. Then the functional calculus Φ constructed in Theorem 2.1 is isometric. For completeness sake we indicate the well-known argument. It suffices to show that the spectral inclusion

$$f(\sigma(S) \cap U) \subset \sigma(\Phi(f))$$

holds for all functions $f \in H^\infty(U)$. Indeed, in this case, we obtain that

$$\|\Phi(f)\| \geq \sup\{|f(z)|;\ z \in \sigma(S) \cap U\} = \|f\|_U$$

for every function $f \in H^\infty(U)$. To prove the above spectral inclusion property, fix a point $w \in U$ such that $f(w) \notin \sigma(\Phi(f))$. It is well known that, since U is a polydomain, there are functions $f_1, \dots, f_n \in H^\infty(U)$ such that

$$f(z) - f(w) = \sum_{\nu=1}^{n} (z_\nu - w_\nu) f_\nu(z) \qquad (z \in U).$$

By applying the functional calculus Φ to both sides, one finds that

$$\Phi(f) - f(w) = \sum_{\nu=1}^{n} (S_\nu - w_\nu)\Phi(f_\nu).$$

Since the left-hand side is invertible, there are operators $R_1, \dots, R_n \in L(H)$ in the commutant of S which satisfy the identity

$$I = \sum_{\nu=1}^{n} (S_\nu - w_\nu) R_\nu.$$

Since the Taylor spectrum $\sigma(S)$ of S is contained in its commutant spectrum (see [10], Lemma 2.2.4), it follows that $w \notin \sigma(S)$.

2.2 Lemma. *Let $S = (S_1, \ldots, S_n) \in L(H)^n$ be a subnormal tuple consisting of pure subnormal operators S_ν $(1 \le \nu \le n)$. Suppose that the functional calculus $\Phi : H^\infty(U) \to L(H)$ of S described in Theorem 2.1 is isometric. Then S is reflexive.*

Proof. We use the notations from the proof of Theorem 2.1. Since by hypothesis the representation

$$\Phi : H^\infty(U) \to L(H), \quad \Phi(f) = \bigoplus_{i \in I} \Phi_i(f|C_i)$$

is isometric, all the functional calculi $\Phi_i : H^\infty(C_i) \to L(H_i)$ are isometric. But then the subnormal tuples $S^{(i)} \in L(H_i)^n$ $(i \in I)$ are completely non-unitary and possess the isometric weak* continuous functional calculi

$$\Phi^{(i)} : H^\infty(\mathbb{D}^n) \to L(H_i).$$

As an application of Corollary 4.5 in [9] the subnormal tuples $S^{(i)}$ are reflexive, and according to Theorem 3.5 in the same paper, the dual algebras $\mathfrak{A}_{S^{(i)}}$ generated by these tuples satisfy the factorization property $(\mathbb{A}_1(16))$. In particular, these dual algebras are WOT-closed and the weak* and weak operator topologies coincide on $\mathfrak{A}_{S^{(i)}}$ $(i \in I)$ (see Proposition 2.09 in [3]).

Since the representations Φ_i and $\Phi^{(i)}$ are weak* continuous and isometric, we obtain that

$$\mathfrak{A}_{(S|H_i)} \subset \mathrm{Im}\Phi_i = \mathrm{Im}\Phi^{(i)} = \mathfrak{A}_{S^{(i)}} = \mathrm{Alg\ Lat}(S^{(i)})$$

for every $i \in I$.

Fix an operator $C \in \mathrm{Alg\ Lat}(S)$. Then it follows that

$$C|H_i \in \mathrm{Alg\ Lat}(S|H_i) \subset \mathrm{Alg\ Lat}(\mathfrak{A}_{S^{(i)}}) = \mathfrak{A}_{S^{(i)}}$$

for all index tuples $i \in I$. Hence $\mathrm{Alg\ Lat}(S)$ is contained in the direct sum $\bigoplus_{i \in I} \mathfrak{A}_{S^{(i)}}$ which by definition consists of all operators $A \in L(H)$ with $H_i \in \mathrm{Lat}(A)$ and $A|H_i \in \mathfrak{A}_{S^{(i)}}$ for all $i \in I$. Since this direct sum inherits property $(\mathbb{A}_1(16))$ (Proposition 2.055 in [3]), a standard argument (Proposition 2.5 in [11]) implies that S is reflexive and thus completes the proof. □

Specializing to the case of dominating spectrum, we obtain the following consequence.

2.3 Corollary. *Let $S = (S_1, \ldots, S_n) \in L(H)^n$ be a subnormal tuple consisting of pure subnormal operators S_ν. If $\sigma(S)$ is dominating in $V = V_1 \times \ldots \times V_n$ for some bounded open sets $V_\nu \subset \mathbb{C}$, then S is reflexive.*

Proof. As before, let us denote by $U = U_1 \times \ldots \times U_n$ the cartesian product of the largest bounded open sets $U_\nu \subset \mathbb{C}$ in which the spectra $\sigma(S_\nu)$ are dominating. Since $\sigma(S_\nu) = \pi_\nu(\sigma(S))$, an elementary argument shows that the sets $\sigma(S_\nu)$ are dominating in V_ν. In particular, it follows that $V \subset U$.

Fix a function $f \in H^\infty(U)$ and a point $z = (z_1, \ldots, z_n) \in U$. Let $\nu \in \{1, \ldots, n\}$ be arbitrary. We claim that there is a sequence $(z_{\nu,j})_{j\geq 1}$ in V_ν such that

$$|f(z_1, \ldots, z_{\nu,j}, \ldots, z_n)| > |f(z)| - \frac{1}{j} \quad (j \geq 1).$$

Indeed, since $\sigma(S_\nu)$ is dominating in U_ν, there is certainly such a sequence in $U_\nu \cap \sigma(S_\nu)$ and since $\sigma(S_\nu) \subset \overline{V}_\nu$, we can also find such a sequence in V_ν. By applying this argument repeatedly, one obtains that $|f(z)| \leq \|f\|_V$ for arbitrary $z \in U$. But then

$$\|f\|_U = \|f\|_V = \|f\|_{V\cap\sigma(S)} \leq \|f\|_{U\cap\sigma(S)}.$$

Hence $\sigma(S)$ is dominating in U, and the assertion follows from Lemma 2.2. □

References

[1] C. Apostol, Functional calculus and invariant subspaces, J. Operator Theory 4 (1980), 159-190.

[2] H. Bercovici, A factorization theorem with applications to invariant subspaces and the reflexivity of isometries, Math. Res. Lett. 1 (1994), 511-518.

[3] H. Bercovici, C. Foias, C. Pearcy, Dual algebras with applications to invariant subspaces and dilation theory, CBMS Regional Conf. Series in Math., vol. 56, Amer. Math. Soc., Providence, RI 1985.

[4] G. Dales, P. Aiena, J. Eschmeier, K. Laursen and G. Willis, Introduction to Banach algebras, operators, and harmonic analysis, London Mathematical Society, Student Texts, 57, Cambridge University Press, Cambridge 2003.

[5] J.A. Deddens, Every isometry is reflexive, Proc. Amer. Math. Soc. 28 (1971), 509-512.

[6] M. Didas, Dual algebras generated by von Neumann n-tuples over strictly pseudoconvex sets. Dissertationes Math. 425, 2004.

[7] M. Didas, Spherical isometries are reflexive, Integral Equations Operator Theory 52 (2005), 599-604.

[8] J. Eschmeier, Algebras of subnormal operators on the unit ball, J. Operator Theory 42 (1999), 37-76.

[9] J. Eschmeier, Algebras of subnormal operators on the unit polydisc, In: Recent Progress in Functional Analysis (eds. K. Bierstedt, J. Bonet, M. Maestre, J. Schmets), pp. 159-171, North-Holland, Amsterdam 2001.

[10] J. Eschmeier, M. Putinar, Spectral decompositions and analytic sheaves, LMS Monograph Series, Vol. 10, Clarendon Press, Oxford 1996.

[11] D. Hadwin, E. Nordgren, Subalgebras of reflexive algebras. J. Operator Theory 7 (1982), 3-23.

[12] W.S. Li and J. McCarthy, Reflexivity of isometries, Studia Math. 124 (1997), 101-105.

[13] R. Olin and J.E. Thomson, Algebras of subnormal operators. J. Funct. Anal. 37 (1980), 271-301.

[14] D. Sarason, Invariant subspaces and unstarred operator algebras, Pacific J. Math. 17 (1966), 511-517.

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