① $n\sqrt{n} \geq n+\sqrt{n}$, $n \geq 4$.

IS $(n+1)\sqrt{n+1} = n\sqrt{n+1} + \sqrt{n+1} > n\sqrt{n} + \sqrt{n+1} \overset{IV}{\geq} n+\sqrt{n}+\sqrt{n+1}$
$> n+1+\sqrt{n+1}$.

② $3^{n+1} + 2^{3n+1}$ ist durch 5 teilbar, $n \geq 1$.

IS $3^{n+2} + 2^{3n+4} = 3\cdot 3^{n+1} + 8\cdot 2^{3n+1} = 3\overbrace{(3^{n+1}+2^{3n+1})}^{IV} + 5\cdot 2^{3n+1}$

③ $\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$, $n \geq 1$.

IS $\sum_{k=0}^{n+1} 2^k = \sum_{k=0}^{n} 2^k + 2^{n+1} \overset{IV}{=} 2^{n+1} - 1 + 2^{n+1} = 2\,2^{n+1} - 1 = 2^{n+2} - 1$.

④ $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$, $n \geq 1$

IS $\sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \sum_{k=1}^{n} \frac{1}{k(k+1)} + \frac{1}{(n+1)(n+2)} \overset{IV}{=} \frac{n}{n+1} + \frac{1}{(n+1)(n+2)}$

$= \frac{n(n+2)+1}{(n+1)(n+2)} = \frac{n^2+2n+1}{(n+1)(n+2)} = \frac{(n+1)^2}{(n+1)(n+2)} = \frac{(n+1)}{(n+2)}$

⑤ $\prod_{k=2}^{n} \left(1 - \frac{1}{k^2}\right) = \frac{n+1}{2n}$, $n \geq 2$.

IS $\prod_{k=2}^{n+1} \left(1 - \frac{1}{k^2}\right) = \prod_{k=2}^{n} \left(1 - \frac{1}{k^2}\right)\left(1 - \frac{1}{(n+1)^2}\right) \overset{IV}{=} \left(\frac{n+1}{2n}\right)\left(1 - \frac{1}{(n+1)^2}\right)$

$= \left(\frac{n+1}{2n}\right) \frac{(n+1)^2 - 1}{(n+1)^2} = \frac{n(n+2)}{2n(n+1)} = \frac{n+2}{2(n+1)}$

⑥ $\sum_{k=1}^{n} \log\left(1+\frac{1}{k}\right) = \log(1+n)$, $n \geq 1$.

IS $\sum_{k=1}^{n+1} \log\left(1+\frac{1}{k}\right) = \sum_{k=1}^{n} \log\left(1+\frac{1}{k}\right) + \log\left(1+\frac{1}{n+1}\right) \overset{IV}{=} \log(1+n) + \log\left(1+\frac{1}{n+1}\right)$
$= \log\left((1+n)\left(1+\frac{1}{n+1}\right)\right) = \log(2+n)$