

Analytical Methods for PDEs (SoSe 2018)

Hometask N 1

Ex. 1 Classify and find the families of characteristics for the equations:

- (a) $u''_{xx} + 4u''_{xy} + 3u''_{yy} + 2u'_x = 0.$
- (b) $u''_{xx} - 2xu''_{xy} + x^2u''_{yy} + u'_x - u'_y + 2u = 0.$
- (c) $u''_{xx} + 4u''_{xy} + 5u''_{yy} + u'_x + u'_y = 0.$

Ex. 2 Find the domains of parabolicity / ellipticity / hyperbolicity for the equations:

- (a) $u''_{xx} + xyu''_{yy} - yu'_x + xu'_y = 0.$
- (b) $(x^2 - 1)u''_{xx} + 2xyu''_{xy} + (y^2 - 1)u''_{yy} = 0.$

Ex. 3 Reduce the equations to a canonical form:

- (a) $2u''_{xx} + 3u''_{xy} + u''_{yy} + 7u'_x + 4u'_y - 2u = 0.$
- (b) $u''_{xx} + 4u''_{xy} + 13u''_{yy} + u'_x = 0.$
- (c) $u''_{xx} + 4u''_{xy} + 4u''_{yy} - 4u'_y = 0.$
- (d) $e^{2x}u''_{xx} + 2e^{x+y}u''_{xy} + e^{2y}u''_{yy} - xu = 0.$
- (e) $2y^2u''_{xx} + 14xyu''_{xy} + 25x^2u''_{yy} = 2\frac{y^2}{x}u'_x + 25\frac{x^2}{y}u'_y.$
- (f) $xu''_{xx} + u''_{yy} = 0, \quad x > 0.$

Ex. 4 Solve the following equations:

- (a) $u''_{xy} = 2(x + y).$
- (b) $u''_{xy} = 2u'_x.$
- (c) $u''_{xy} = 2xu'_y.$
- (d) $u''_{xx} + u'_x = y.$

Ex. 5 Reduce to a canonical form and solve the equations:

- (a) $u''_{xx} - 8u''_{xy} + 12u''_{yy} = 0.$
- (b) $4u''_{xx} + 12u''_{xy} + 9u''_{yy} = 0.$
- (c) $x^2u''_{xx} - y^2u''_{yy} - 2yu'_y = 0.$