UNIVERSITÄT DES SAARLANDES
Fakultät für Mathematik und Informatik
Fachrichtung Mathematik
Dr. Yana Kinderknecht

UNIVERSITÄT DES SAARLANDES

Analytical Methods for PDEs (SoSe 2018)

Hometask N 2

Ex. 6 Solve the following Cauchy problems:

(a) $u''_{tt} - 4u''_{xx} = 0, \quad u(0,x) = 0, \quad u'_t(0,x) = \sin x.$

(b) $u''_{tt} - 9u''_{xx} = 0, \quad u(0,x) = e^{-x^2}, \quad u'_t(0,x) = \frac{1}{1+x^2}.$

Remark: (a) if φ and ψ are odd functions then the solution of the Cauchy problem (1) satisfies the condition $u(t,0) = 0$ (Dirichlet condition at $x = 0$);

(b) if φ and ψ are even functions then the solution of the Cauchy problem (1) satisfies the condition $u'_x(t,0) = 0$ (Neumann condition at $x = 0$).

Here

$$\begin{cases} u''_{tt} - a^2u''_{xx} = 0, & x \in \mathbb{R}, \quad t > 0, \\ u(0,x) = \varphi(x), & x \in \mathbb{R}, \\ u'_t(0,x) = \psi(x), & x \in \mathbb{R}. \end{cases} \tag{1}$$

Vibrations of a half-infinite string ($a = 1, \ 0 \leqslant x < \infty$) can be described by the following initial-boundary value problem (with some α, β, ν):

$$\begin{cases} u''_{tt} - u''_{xx} = 0, & x \in (0,\infty), \quad t > 0, \\ u(0,x) = \phi(x), & x \in (0,\infty), \\ u'_t(0,x) = \psi(x), & x \in (0,\infty), \\ \alpha u(t,0) + \beta u'_x(t,0) = \nu(t). \end{cases} \tag{2}$$

In particular, the case of a string with the fixed end corresponds to the Dirichlet boundary condition $u(t,0) = 0$; the case of a string with the free end — to the Neumann boundary condition $u'_x(t,0) = 0$.

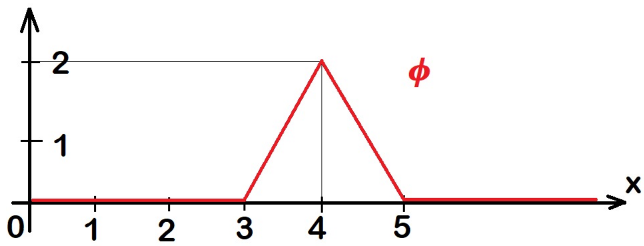
Ex. 7 Draw the profile of a half-infinite string at the moments of time $t_0 = 0$, $t_1 = 0,5$, $t_2 = 1$, $t_3 = 2$, $t_4 = 3$, $t_5 = 3,5$. $t_6 = 4$, $t_7 = 4,5$, $t_8 = 5$, $t_9 = 6$ for the following cases:

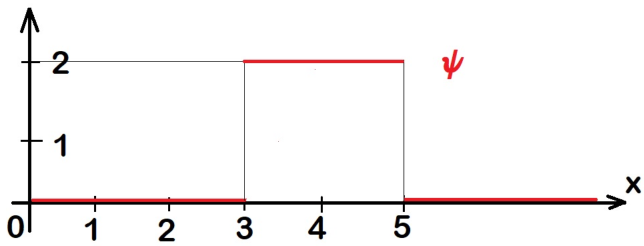
(a) ϕ is given below, $\psi \equiv 0$, $u(t,0) = 0$;

(b) ϕ is given below, $\psi \equiv 0$, $u'_x(t,0) = 0$;

(c) $\phi \equiv 0$, ψ is given below, $u(t,0) = 0$;

(d) $\phi \equiv 0$, ψ is given below, $u'_x(t,0) = 0$;





Hint: Reduce the considered initial-boundary value problems to the appropriate Cauchy problems for an infinite string by extending the initial conditions in a suitable way (making them odd or even functions).

Ex. 8 Solve the following initial-boundary value problems ($t \geqslant 0$, $x \geqslant 0$):

(a) $u''_{tt} - u''_{xx} = 0$, $u(0,x) = x^3$, $u'_t(0,x) = \cos x$, $u'_x(t,0) = 0$.

(b) $u''_{tt} - 16u''_{xx} = 0$, $u(0,x) = 0$, $u'_t(0,x) = xe^{-x^2}$, $u(t,0) = 0$.