UNIVERSITÄT DES SAARLANDES
Fakultät für Mathematik und Informatik
Fachrichtung Mathematik
Dr. Yana Kinderknecht

Analytical Methods for PDEs (SoSe 2018)

Hometask N 5

Ex. 14 Let $\Omega \subset \mathbb{R}^2$ be the circle of radius R with center at the origin. Solve the following boundary-value problems:

(a) $\Delta u = 0$ in Ω, $\quad u(x,y) = A + Bx$ on $\partial\Omega$.

(b) $\Delta u = 0$ in Ω, $\quad u(x,y) = Axy$ on $\partial\Omega$.

(c) $\Delta u = 0$ in Ω, $\quad \frac{\partial u}{\partial r}\Big|_{r=R} = A\cos\varphi$.

Ex. 15 Let $\Omega \subset \mathbb{R}^2$ be the complement of the circle of radius 1 with center at the origin. Solve the following boundary-value problems:

(a) $\Delta u = 0$ in Ω, $\quad u\Big|_{r=1} = \cos^2\varphi$.

(b) $\Delta u = 0$ in Ω, $\quad u(x,y) = Axy$ on $\partial\Omega$.

(c) $\Delta u = 0$ in Ω, $\quad \frac{\partial u}{\partial r}\Big|_{r=1} = A\cos\varphi$.

Ex. 16 Let $\Omega \subset \mathbb{R}^2$ be the ring between circles with radius 1 and 2 with center at the origin. Solve the following boundary-value problems:

(a) $\Delta u = 0$ in Ω, $\quad u\Big|_{r=1} = u_1 \equiv \text{const}, \quad u\Big|_{r=2} = u_2 \equiv \text{const}$.

(b) $\Delta u = 0$ in Ω, $\quad \frac{\partial u}{\partial r}\Big|_{r=1} = 0, \quad u\Big|_{r=2} = \cos\varphi$.

(c) $\Delta u = 0$ in Ω, $\quad u\Big|_{r=1} = 0, \quad \frac{\partial u}{\partial r}\Big|_{r=2} = A\sin 2\varphi$.

Ex. 17 Let $\Omega \subset \mathbb{R}^2$ be the sector $0 < r < R$, $0 < \varphi < l$. Solve the following boundary-value problems for the Laplace equation in Ω:

(a) $l := \frac{\pi}{3}, \quad u(r,0) = u(r,\pi/3) = 0, \quad u(R,\varphi) = \sin 6\varphi$.

(b) $l := \frac{\pi}{2}, \quad u(r,0) = u(r,\pi/2) = 0, \quad u(R,\varphi) = \varphi$.

(c) $l := \frac{\pi}{4}$, $\frac{\partial u}{\partial \varphi}(r, 0) = u(r, \pi/4) = 0$, $u(R, \varphi) = \cos 2\varphi$.

Ex. 18 Let $\Omega \subset \mathbb{R}^2$ be the circle of radius r_0 with center at the origin. Solve the following initial-boundary value problem for the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad (x, y) \in \Omega, \ t > 0$$
$$u\Big|_{r=r_0} = 0,$$
$$u\Big|_{t=0} = J_0 \left(\frac{\mu}{r_0} r \right), \quad \mu := \mu_0^0 \text{ is the first zero of } J_0,$$
$$\frac{\partial u}{\partial t}\Big|_{t=0} = 0.$$

Ex. 19 Let $\Omega \subset \mathbb{R}^2$ be the circle of radius 1 with center at the origin. Solve the following boundary value problem for the Helmholtz equation:

$$\Delta u + 4u = 0, \quad \text{in } \Omega,$$
$$u\Big|_{r=1} = 11 \sin 3\varphi.$$