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Analytical Methods for PDEs (SoSe 2018)

Hometask N 7

Ex. 25 Using the properties of the Fourier transform and the results of the Hometask N 6, find the Fourier transforms of the following functions (without calculating integrals):

a) $f(x) = \frac{1}{x^2+1}$,

b) $f(x) = \frac{x}{x^2+1}$,

c) $f(x) = \frac{x-2}{x^2+4}$,

d) $f(x) = \frac{2x+1}{(x^2+2x+2)(x^2+1)}$,

e) $f(x) = \arctan\, x - \arctan(x+1)$,

f) $f(x) = \sin x\,(\arctan\, x - \arctan(x+1))$

g) $f(x) = \frac{\sin x}{x^2+1}$,

h) $f(x) = \frac{\cos x}{x^2+4}$,

i) $f(x) = xe^{-x}\theta(x)$,

j) $f(x) = (x\sin x)e^{-x}\theta(x)$,

k) $f(x) = xe^{-|x|}$,

l) $f(x) = (1 + a|x|)e^{-a|x|}$, $a > 0$ is a fixed constant,

m) $f(x) = e^{ax}\theta(-x)$, $a > 0$ is a fixed constant,

n) $f(x) = \frac{\sin ax}{x}$,

o) $f(x) = (x+7)e^{-|x+7|}$.

Which space belong these functions to? ($L^1(\mathbb{R})$ or $L^2(\mathbb{R})$)

Ex. 26 Find the Fourier transform of the following functions:

a) $f(x) = e^{-x^2} * \frac{1}{x^2+1}$,

b) $f(x) = e^{-|x|} * \frac{\cos x}{x^2+1}$,

c) $f(x) = e^{-|x-2|} * \frac{\sin x}{x^2+4}$,

d) $f(x) = e^{-ax^2} * (-2axe^{-ax^2}) * \frac{1}{x^2+1}$, $a > 0$.

Ex. 27 Consider the following initial–boundary value problem for the one-dimensional heat equation ($a > 0$):

$$\begin{cases} \frac{\partial u}{\partial t}(t,x) = a^2 \frac{\partial^2 u}{\partial x^2}(t,x), & t > 0, \quad x > 0, \\ u(0,x) = u_0(x), & x > 0, \\ \alpha u(t,0) + \beta \frac{\partial u}{\partial x}(t,0) = 0, & t > 0. \end{cases}$$

Show that

a) $u(t,x) := (4a^2\pi t)^{-1/2} \int\limits_0^\infty u_0(y) \left(e^{-\frac{|x-y|^2}{4a^2 t}} - e^{-\frac{|x+y|^2}{4a^2 t}} \right) dy$ solves the problem in the case $\alpha = 1$, $\beta = 0$;

b) $u(t,x) := (4a^2\pi t)^{-1/2} \int\limits_0^\infty u_0(y) \left(e^{-\frac{|x-y|^2}{4a^2 t}} + e^{-\frac{|x+y|^2}{4a^2 t}} \right) dy$ solves the problem in the case $\alpha = 0$, $\beta = 1$.

Hint: Extend u_0 properly (as odd / even function) to the whole $\mathbb{R}$.