Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
Dr. Darya Apushkinskaya

,t

PDE and Boundary-Value Problems (Winter Term 2013/2014)
Assignment H2 - Homework

Problem 2.1 (Formulation of IBVP - 4 Points)

Suppose a metal rod laterally insulated has an initial temperature of $20°C$ but immediately thereafter has one end fixed at $50°C$. The rest of the rod is immersed in a liquid solution of temperature $30°C$. What would be the IBVP that describes this problem?

Problem 2.2 (Derivation of the diffusion equation - 6 Points)

Suppose $u(x,t)$ measures the concentration of a substance in a moving stream (moving with velocity ν). Suppose the concentration $u(x,t)$ changes both by diffusion and convection; derive the equation

$$u_t = \alpha^2 u_{xx} - \nu u_x$$

from the fact that at any instant time, the total mass of the material is not created or destroyed in the region $[x, x + \Delta x]$.

HINT: Write the conservation equation

$$\text{Change of mass inside } [x, x+\delta x] = \text{Change due to } \textit{diffusion} \text{ across the boundaries}$$
$$+ \text{ Change due to the material being } \textit{carried} \text{ across the boundaries.}$$

Problem 2.3 (Solving IBVP - 4x3=12 Points)

a) What is the solution to the IBVP

$$\text{PDE:} \quad u_t = u_{xx}, \qquad 0 < x < 1, \quad 0 < t < \infty$$

$$\text{BCs:} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 0 \end{cases}, \qquad 0 < t < \infty$$

$$\text{IC:} \quad u(x,0) = 1, \qquad 0 \leqslant x \leqslant 1$$

(Note that this problem is physically impossible, since we are pulling the temperature from 1 to 0 instantaneously. In most problems, if the BCs are zero, then the initial themperature $\phi(x)$ should also be zero at $x = 0$ and $x = 1$).

b) What is the solution to problem a) if the IC is changed to

$$u(x, 0) = \sin(2\pi x) + \frac{1}{3}\sin(4\pi x) + \frac{1}{5}\sin(6\pi x)?$$

c) What is the solution to problem a) if the IC is changed to

$$u(x, 0) = x - x^2?$$

Problem 2.4 (Transformation of IBVP - 6 Points)

Transform

$$\text{PDE:} \quad u_t = u_{xx}, \qquad 0 < x < 1, \quad 0 < t < \infty$$

$$\text{BCs:} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 1 \end{cases}, \qquad 0 < t < \infty$$

$$\text{IC:} \quad u(x, 0) = x^2, \qquad 0 \leqslant x \leqslant 1$$

to zero BCs and solve the new problem. What is the steady-state solution?

Deadline for submission: Wednesday, November 13, 10:10 am