Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
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Calculus of Variations (Summer Term 2014)
Assignment H5 - Homework

Problem 4.1 (9 Points)

Minimize the functional

$$J[y] = \int_{-1}^{1} y\sqrt{1 + (y')^2}dx$$

subject to the end-point conditions that $y(-1) = 2$ and $y(1) = 2$. Use the modification of the Ritz Method that we discussed in Lecture 13. Compare the obtained result with the result from Lecture 13.

Hint:

1. Approximate a curve by a polynomial

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$$

2. Calculate $J[a_i]$ and corresponding derivatives $\frac{\partial J}{\partial a_i}$.
3. Calculate (using Maple) the roots a_i provided $\frac{\partial J}{\partial a_i} = 0$.

Problem 5.2 (9 Points)

Use Ritz's method to find an approximate, non-trivial solution to the differential equation

$$y'' + \frac{1}{x}y' + \lambda y = 0$$

in the domain $x \in [0, 1]$ where $y(0)$ is non-singular and $y(1) = 1$, and hence determine an approximate value of λ that has a solution.

(*Hint:* Note that the equation can be written in the form

$$\frac{d}{dx}(xy') + \lambda xy = 0$$

and find the corresponding integral for which this is the Euler-Lagrange equation.

Once you have a variational problem, use the trial function

$$y_{trial} = a + bx^2 + cx^4,$$

which we have chosen because the solution is expected to be an even function.)

Problem 5.3 (8 Points)

How could we minimize the functional

$$J[y] = \int_0^1 \left(y^2 + y'^2\right) dx, \qquad y(0) - 0, \quad y(1) = 1$$

by the method of Ritz?

(*Hint:* If we introduce a new function $z(t)$

$$z(t) = (1 - x)\, y(t)$$

we note that $z(t)$ satisfies $z(0) = 0$ and $z(1) = 0$.)

Deadline for submission: Wednesday, July 02, 12 am