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Mainardi–Wright and Mittag–Leffler special functions as well as their connections to generalized grey Brownian motion

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We consider the Mainardi–Wright special function M_ν, $\nu \in (0,1)$, discuss some of its properties (an integral representation and a formula for "absolute moments") and show that $M_\nu : (0,\infty) \to \mathbb{R}$ is a probability density function (PDF). Therefore, M_ν allows to construct the probability density function

$$f_{\alpha,\beta}(t,x) := \frac{1}{2} t^{-\alpha/2} M_{\beta/2}\left(\frac{|x|}{t^{\alpha/2}}\right), \qquad x \in \mathbb{R},$$

for each $\beta \in (0,1]$, $\alpha \in (0,2)$ and $t > 0$. We consider also the Mittag–Leffler special function E_ν, $\nu \geq 0$, and establich its connection to the Mainardi–Wright function M_ν via the Fourier transform. The function E_ν is needed further to define a family of stochastic processes $(X_{\alpha,\beta}(t))_{t\geq 0}$, $\beta \in (0,1]$, $\alpha \in (0,2)$, known as "Generalized Grey Brownian Motion" (GGBM). The GGBM is introduced in 2008 (by Mainardi and his coauthors) and coincides with standard Brownian motion when $\alpha = \beta = 1$, with fractional Brownian motion when $\beta = 1$, $\alpha \in (0,2)$, with grey Brownian motion (introduced by Schneider in 1988 in terms of his "Grey Noise Analysis") when $\alpha = \beta \neq 1$. We show that $f_{\alpha,\beta}(t,x)$ is the marginal density function of $(X_{\alpha,\beta}(t))_{t\geq 0}$. Moreover, $(X_{\alpha,\beta}(t))_{t\geq 0}$ is an $\frac{\alpha}{2}$–self-similar process with stationary increments. Finally, we present a particular realization of GGBM.