

Discrete-Time Mathematical Finance

Assignment sheet 2

Exercise 1 (2+2 points)

Let \mathcal{M} be a finite market with $T = 2$, $D = 1$, $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\mathcal{F}_1 = \{\emptyset, \{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \Omega\}$, $S_t^0 = 1$ for $t = 0, 1, 2$ and

$$\begin{aligned}(S_0^1, S_1^1, S_2^1)(\omega_1) &= (1, 1, 1), & (S_0^1, S_1^1, S_2^1)(\omega_2) &= (1, 1, 2), \\ (S_0^1, S_1^1, S_2^1)(\omega_3) &= (1, 2, 2), & (S_0^1, S_1^1, S_2^1)(\omega_4) &= (1, 2, 3).\end{aligned}$$

- (a) Find a perfect hedge $\varphi \in \mathcal{A}^{sf}$ for the Put option $\text{Put}(44, 2, 1)$ and compute $V_0(\varphi)$.
- (b) Repeat part (a) for the Put option $\text{Put}(2, 2, 1)$.

Hint: The hedge for the Put option in part (a) can be seen immediately. For part (b) you have to work backwards in time starting from the maturity 2 and need to solve three 2×2 systems of linear equations.

Exercise 2 (2+2 points)

Let \mathcal{M} be a finite market with $T = 1$.

- (a) Consider specifically the market in Example 1.2.4. Show that each contract with maturity 1 can be replicated by a self-financing portfolio. (In such situations, the market is called *complete*.)
- (b) Now let \mathcal{M} be a finite market with $N > D + 1$. Show that there exist contracts which are not replicable.