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Discrete-Time Mathematical Finance

Assignment sheet 3

Exercise 1   (3 points)

Consider the market in Exercise 1 of Assignment sheet 2. Show that LOP holds in this market and that there exists arbitrage.

Exercise 2   (2+1+1+1 points)

Consider a one-period-model $\mathcal{M}$, where $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $D = 1$, $\mathcal{F}_0 = \{\emptyset, \Omega\}$, $\mathcal{F}_1 = 2^\Omega$, $S_t^0 = 1$ for $t = 0, 1$ and

$$S_0^1 = 12, \ S_1^1(\omega_1) = 15, \ S_1^1(\omega_2) = 9, \ S_1^1(\omega_3) = 6.$$

(a) Compute all linear price systems in $\mathcal{M}$.

(b) Compute $\mathcal{I}_\xi := \{\pi(\xi), \ \pi \text{ linear price system}\}$ for $\xi = \text{Call}(7, 1, 1)$.

(c) Show that, for any $\xi \in L_0(\mathcal{F}_1)$, there exist real numbers $\alpha_0, \alpha_1, \alpha_2$ such that

$$\xi = \alpha_0 S_1^0 + \alpha_1 S_1^1 + \alpha_2 \text{Call}(7, 1, 1).$$

(d) Conclude from (b) and (c):

$$\xi \in L_0(\mathcal{F}_1) \setminus \mathcal{H} \iff \mathcal{I}_\xi \text{ is an open interval.}$$