Prof. Dr. Christian Bender
M.Sc. Christian Gärtner

Saarland University, Winter Term 2017/18
December 1, 2017

Discrete-Time Mathematical Finance

Assignment sheet 6

Exercise 1 (4 points)

Let Ω be a finite sample space and let $\mathcal{F}$ be a σ-algebra on Ω. Show that the atom system of $\mathcal{F}$ is unique.

Exercise 2 (4 points)

Let $\mathcal{M} = (\Omega, \mathcal{F}, P, (S_t)_{t\in\{0,1,2\}}, (\mathcal{F}_t)_{t\in\{0,1,2\}}, \mathcal{A}^{sf})$ be a market with two assets and $\Omega = \{\omega_1, \dots, \omega_7\}$. Let $S_t^0 = 100 + 5t$ for $t = 0, 1, 2$ and

$$\begin{aligned}
&S_0^1 = 100,\\
&S_1^1(\omega_1) = S_1^1(\omega_2) = S_1^1(\omega_3) = 110,\ S_1^1(\omega_4) = S_1^1(\omega_5) = S_1^1(\omega_6) = 90, S_1^1(\omega_7) = 63;\\
&S_2^1(\omega_1) = 120,\ S_2^1(\omega_2) = 110,\\
&S_2^1(\omega_3) = S_2^1(\omega_4) = 100,\ S_2^1(\omega_5) = 90,\ S_2^1(\omega_6) = 75,\ S_2^1(\omega_7) = 66.
\end{aligned}$$

Moreover, let

$$\mathcal{F}_0 = \{\Omega, \emptyset\},\ \mathcal{F}_1 = \{(S_1^1)^{-1}(B);\ B \text{ Borel set in } \mathbb{R}\},\ \mathcal{F}_2 = 2^\Omega.$$

Decompose $\mathcal{M}$ into 1-period-submodels and check if these are arbitrage-free.