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Discrete-Time Mathematical Finance

Assignment sheet 7

Exercise 1 (8 points)

Let $\mathcal{M} = (\Omega, \mathcal{F}, P, (S_t)_{t\in\{0,1,2\}}, (\mathcal{F}_t)_{t\in\{0,1,2\}}, \mathcal{A}^{sf})$ be a market with two assets and $\Omega = \{\omega_1, \dots, \omega_4\}$. Let $S_t^0 = 100$ for $t = 0, 1, 2$ and

$$\begin{aligned}
&S_0^1 = 100,\\
&S_1^1(\omega_1) = S_1^1(\omega_2) = 110,\ S_1^1(\omega_3) = S_1^1(\omega_4) = 90\\
&S_2^1(\omega_1) = 120,\ S_2^1(\omega_2) = 100,\\
&S_2^1(\omega_3) = 100,\ S_2^1(\omega_4) = 80
\end{aligned}$$

Moreover, let

$$\mathcal{F}_0 = \{\Omega, \emptyset\},\ \mathcal{F}_1 = \{(S_1^1)^{-1}(B);\ B \text{ Borel set in } \mathbb{R}\},\ \mathcal{F}_2 = 2^\Omega.$$

(a) Find an equivalent martingale measure for $\mathcal{M}$.

(b) Find a perfect hedge for an Asian Call option on S^1 with strike 90 and payoff

$$\xi = \left(\frac{1}{3}(S_0^1 + S_1^1 + S_2^1) - 90\right)^+$$

and compute the hedging price.

(c) Repeat part (b) for a floating-strike lookback Call option on S^1 and payoff

$$\eta = \left(S_2^1 - \min_{t=0,1,2} S_t^1\right).$$