Prof. Dr. Christian Bender
M.Sc. Christian Gärtner

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Discrete-Time Mathematical Finance

Assignment sheet 8

Exercise 1 (4 points)

Let $\mathcal{M} = (\Omega, \mathcal{F}, P, (S_t)_{t\in\{0,1,2\}}, (\mathcal{F}_t)_{t\in\{0,1,2\}}, \mathcal{A}^{sf})$ be a market with two assets and 5 states, $\Omega = \{\omega_1, \dots, \omega_5\}$. Let $S_t^0 = 1$ for $t = 0, 1, 2$ and

$$\begin{aligned}
&S_0^1 = 10, S_1^1(\omega_1) = S_1^1(\omega_2) = 11,\\
&S_1^1(\omega_3) = S_1^1(\omega_4) = S_1^1(\omega_5) = 9,\\
&S_2^1(\omega_1) = 12,\ S_2^1(\omega_2) = S_2^1(\omega_3) = 10,\\
&S_2^1(\omega_4) = 9,\ S_2^1(\omega_5) = 8.
\end{aligned}$$

Moreover, let

$$\mathcal{F}_0 = \{\Omega, \emptyset\},\ \mathcal{F}_1 = \{(S_1^1)^{-1}(B);\ B \text{ Borel set in } \mathbb{R}\},\ \mathcal{F}_2 = 2^\Omega.$$

(a) Show that an equivalent martingale measure for this model is given by $Q : 2^\Omega \to [0,1]$ with $Q(\{\omega_1\}) = Q(\{\omega_2\}) = \frac{1}{4}$, $Q(\{\omega_3\}) = \frac{1}{12}$ and $Q(\{\omega_4\}) = \frac{1}{3}$.

(b) Consider the contract $\xi = 2Call(9,2,1) - 3Put(10,2,1)$. Compute the fair price process $V_t^{\xi,Q}$ for $t = 0,1,2$.

Exercise 2 (4 points)

Let $\mathcal{M} = (\Omega, \mathcal{F}, P, (S_t)_{t\in\{0,1,2\}}, (\mathcal{F}_t)_{t\in\{0,1,2\}}, \mathcal{A}^{sf})$ be a market with two assets and 6 states, $\Omega = \{\omega_1, \dots, \omega_6\}$. The value process of the assets is given by $S_t^0 = 1$, $t = 0,1,2$ as well as

$$\begin{aligned}
S_0^1 &= 100,\ S_1^1(\omega_1) = S_1^1(\omega_2) = S_1^1(\omega_3) = 110,\ S_1^1(\omega_4) = S_1^1(\omega_5) = S_1^1(\omega_6) = 90;\\
S_2^1(\omega_1) &= 120,\ S_2^1(\omega_2) = 110,\ S_2^1(\omega_3) = 105,\ S_2^1(\omega_4) = 100,\ S_2^1(\omega_5) = 90,\ S_2^1(\omega_6) = 75.
\end{aligned}$$

Moreover, let

$$\mathcal{F}_0 = \{\Omega, \emptyset\},\ \mathcal{F}_1 = \{(S_1^1)^{-1}(B);\ B \text{ Borel set in } \mathbb{R}\},\ \mathcal{F}_2 = 2^\Omega.$$

Find all equivalent martingale measures for this model.