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Stochastics II

1. Tutorial

Exercise 1 **(5 Points)** Let $\Omega = [0,1)$, $\mathcal{F}$ be the σ-field of Borel sets on the interval $[0,1)$ and P be the Lebesgue measure on $[0,1)$. Furthermore let $\mathcal{G}$ be the σ-field, generated by the intervals

$$\left[0, \frac{1}{2}\right), \left[\frac{1}{2}, 1\right)$$

and Y be a discrete random variable defined by

$$Y(x) = i \text{ für } x \in \left[\frac{i}{4}, \frac{i+1}{4}\right), i = 0, \dots, 3.$$

Define the random variable

$$\forall x \in \Omega : \, \xi(x) = x^2$$

on Ω and compute

(i) the conditional expectation $E(\xi|\mathcal{G})$,

(ii) the conditional expectation $\eta = E(\xi|Y)$,

(iii) the conditional expectation $E(\eta|\mathcal{G})$.

Exercise 2 **(5 Points)** Let $\Omega = [0,1)$, $\mathcal{F}$ be the σ-field of Borel sets on the interval $[0,1)$ and P be the Lebesgue measure on $[0,1)$. Find $E(\xi|\eta)$ if

$$\xi(x) = 2x^2 \text{ and } \eta(x) = \begin{cases} 2x & \text{if } x \in \left[0, \frac{1}{2}\right) \\ 2x - 1 & \text{if } x \in \left[\frac{1}{2}, 1\right). \end{cases}$$

Exercise 3 **(5 Points)**

Let $(\xi, \eta_1, \dots, \eta_n)$, $n \in \mathbb{N}$ be an $(n+1)$-variate normal distributed random vector, where the random variables $\eta_1, \dots, \eta_n$ are independent. Calculate

(i) the conditional expectation $E[\xi|\eta_1, \dots, \eta_n]$.

(ii) the conditional variance $E[(\xi - E[\xi|\eta_1, \dots, \eta_n])^2|\eta_1, \dots, \eta_n]$.

Hint: Use the random variable

$$(\xi - E[\xi]) - \sum_{k=1}^{n} \frac{\mathrm{Cov}(\xi, \eta_k)}{\mathrm{Var}(\eta_k)}(\eta_k - E[\eta_k])$$

and show that it is independent of $\eta - E[\eta]$, where $\eta := (\eta_1, \dots, \eta_n)^T$.