Prof. Dr. Christian Bender
M.Sc. Steffen Meyer

Saarland University
WiSe 2019/2010

31st October 2019

Stochastics II

3. Tutorial

Exercise 1 **(5 Points)** Let X be an integrable random variable on a probability space $(\Omega, \mathcal{F}, P)$ and let $\mathcal{G}$, $\mathcal{H} \subset \mathcal{F}$ be two σ-fields. Furthermore we assume that $\sigma(\sigma(X) \cup \mathcal{H})$ is independent of $\mathcal{G}$. Show that the following equation holds true:

$$E[X|\sigma(\mathcal{G} \cup \mathcal{H})] = E[X|\mathcal{H}].$$

Hint: You may assume that X is nonnegative (why?). In this case, one can first show that

$$E[X\mathbb{1}_{A\cap B}] = E[E[X|\mathcal{H}]\mathbb{1}_{A\cap B}] \quad A \in \mathcal{G}, B \in \mathcal{H}$$

and extend this identity in a suitable way.

Exercise 2 **(4 Points)** Let $X = (X_t)_{t\in[0,\infty)}$ and $Y = (Y_t)_{t\in[0,\infty)}$ be real-valued processes with P-a.s. rightcontinuous paths. Furthermore X and Y are modifications of each other. Show that X and Y are indistinguishable.

Exercise 3 **(7 Points)** Let $X := (X_t)_{t\in[0,\infty)}$ be a real-valued stochastic process and $(\mathcal{F}_t^X)_{t\in[0,\infty)}$ the filtration generated by X. Show that $X_t - X_s$ is independent of $\mathcal{F}_s^X$ for all $0 \leq s < t$ if and only if X has independent increments, i.e. the family $(X_{t_k} - X_{t_{k-1}})_{k=1,\ldots,n}$, where $X_{t_0} := 0$ is independent for all $n \in \mathbb{N}$ and $0 \leq t_1 < \ldots < t_n \in [0, \infty)$.