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Algebraic Geometry Summer Term 2018

Exercise Sheet 11. Hand in by Friday, July 6.

Exercise 1. Using the refined Noether Normalization and the Going down theorem, prove that the polynomial ring $K[x_1, \dots, x_n]$ in n variables has Krull dimension n. In fact any maximal chain of prime ideals has length n.

Exercise 2. Compute a primary decomposition of the monomial ideal

$$(xy^2, xyz, z^2w) \subset K[w, x, y, z].$$

Design an algorithm, which computes a irrundandent primary decomposition of a monomial ideal.

Exercise 3. Let $R = K[y_1, \dots, y_d] \hookrightarrow S = K[x_1, \dots, x_n]/I$ be a Noether normalization, such that S is a free R-module. Prove that every associated prime of I has the same codimension $n - d$. In particular there are no embedded primes. One says I is unmixed.

Exercise 4.
Let $R = K[x_1, \dots, x_n]/I$ be a finitely generated commutative K-algebra, and $R_{red} = K[x_1, \dots, x_n]/rad(I)$. Prove:

(1) $R_{red} \cong R/rad(0)$, where $rad(0)$ denotes the radical of $(0) \subset R$.
(2) $\dim R = \dim R_{red}$.