UNIVERSITÄT DES SAARLANDES
Fachrichtung Mathematik
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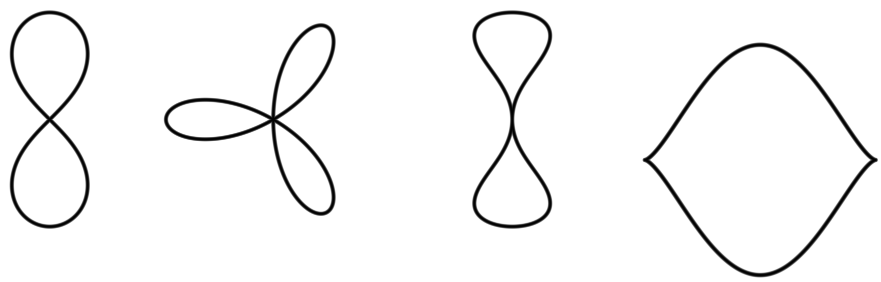
Algebraic Geometry   Summer Term 2018

Exercise Sheet 8. Hand in by Friday, June 15.

Exercise 1 The following picture shows plane curves with different types of singularities:



node   triple point   tacnode   cusps

The curves are defined by the polynomials below:

$$(a)\, y^2 = (1-x^2)^3, \quad (b)\, y^2 = x^2 - x^4,$$
$$(c)\, y^3 - 3x^2y = (x^2+y^2)^2, \quad (d)\, y^2 = x^4 - x^6.$$

Which curve corresponds to which polynomial?

Exercise 2

Let R be a ring, and let M be an R-modules. TFAE

(1) $M = 0$
(2) $M_{\mathfrak{p}} = 0$ for evey prime ideal $\mathfrak{p} \in R$.
(3) $M_{\mathfrak{m}} = 0$ for every maximal ideal $\mathfrak{m} \in R$.

Exercise 3

Let R be a ring, let M', M and M'' be R-modules, and let $M' \to M$ and $M \to M''$ be are R-module homomorphism. TFAE

(1) $0 \to M' \to M \to M'' \to 0$ is a short exact sequence.
(2) $0 \to M'_{\mathfrak{p}} \to M_{\mathfrak{p}} \to M''_{\mathfrak{p}} \to 0$ is a short exact sequence for every prime ideal $\mathfrak{p} \in R$.

(3) $0 \to M'_{\mathfrak{m}} \to M_{\mathfrak{m}} \to M''_{\mathfrak{m}} \to 0$ is a short exact sequence for every maximal ideal $\mathfrak{m} \in R$.

Exercise 4

Compute the intersection multiplicities at $p = (0,0) \in \mathbb{A}^2$ of each pair of curves from Exercise 1.