UNIVERSITÄT DES SAARLANDES
Fachrichtung 6.1 - Mathematik
Prof. Dr. Frank-Olaf Schreyer
Christian Bopp

Exercises Algebraic Geometry

Winterterm 2016/17

The solutions are collected on Tuesday, before the exercise session.
All further informations concerning the lecture can be found here: https://www.math.uni-sb.de/ag/schreyer/index.php/teaching

Sheet 11 — 23.01.2017

Exercise 1 (6.2.5)**.** Let $d \geq 2$, and consider the image C of the parametrization

$$\mathbb{A}^1 \to \mathbb{A}^d,\ t \mapsto (t, t^2, \ldots, t^d).$$

The projectice closure $\overline{C} \subset \mathbb{P}^d$ is known as the **rational normal curve** in $\mathbb{P}^d$. Note that for $d = 2, 3$, we get an irreducible conic respectively the twisted cubic curve. In general, show that $\mathrm{I}(\overline{C})$ is generated by $\binom{d}{2}$ quadrics, and that there is no set of generators with fewer elements. Note that for $d \geq 3$, the number of generators is strictly larger than the codimension $d - 1$.

Exercise 2 (6.3.13)**.** Let A and B be quasi-affine or quasi-projective algebraic sets, and let $\varphi : A \to B$ be a map. Show that φ is a morphism iff the following two conditions hold:

(1) φ is continous.
(2) For any open subset $U \subset B$ and any regular function f on U, the composition $f \circ \varphi$ is a regular function on the open subset $\varphi^{-1}(U) \subset A$.

Exercise 3 (6.3.16)**.** Show that $A = \mathbb{A}^2 \setminus \{(0, 0)\}$ is a quasi-projective algebraic set which is neither projective nor affine.
Hint. To exclude that A is affine, compute the ring $\mathcal{O}(A)$.

Exercise 4 (6.3.21)**.** Show that $\rho_{n,d}$ is a closed embedding for every n and d. Moreover, show that the vanishing ideal of the image is generated by quadrics which are binomials. How many quadrics do you get?