UNIVERSITÄT DES SAARLANDES
Fachrichtung 6.1 - Mathematik
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Exercises Algebraic Geometry

Winterterm 2016/17

The solutions are collected on Tuesday, before the exercise session.
All further informations concerning the lecture can be found here: https://www.math.uni-sb.de/ag/schreyer/index.php/teaching

Sheet 6

05.12.2016

Exercise 1 (3.2.26)**.** Show that the following rings are integral domains, and find their normalizations:

(1) The coordinate ring of the plane curve $\mathrm{V}(y^2 - x^{2k+1}) \subset \mathbb{A}^2$, where $k \geq 1$.
(2) The coordinate ring of the Whitney umbrella $\mathrm{V}(x^2 - y^2 z) \subset \mathbb{A}^3$. □

Exercise 2 (3.3.10)**.** Let $A \subset \mathbb{A}^n$ be an algebraic set. Show that A is a hypersurface iff it is equidimensional of dimension $n-1$.

Exercise 3 (3.4.4)**.** Let $I \subset S = \Bbbk[x_1, \dots, x_4]$ be the ideal which is generated by the 2×2 minors of the matrix

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_4 \end{pmatrix}.$$

Find a Noether normalization as in Theorem 3.4.3.

Exercise 4 (4.1.5)**.** (1) Find all singular points of the curve

$$\mathrm{V}(x^2 - 2x^3 + x^4 + y^2 - 2y^3 + y^4 - \frac{3}{2}x^2y^2) \subset \mathbb{A}^2(\mathbb{C}).$$

Draw a picture of the real points of this curve.

(2) Find all singular points of the curve $\mathrm{V}(f) \subset \mathbb{A}^2(\mathbb{C})$, where

$$\begin{aligned} f = {} & 11\,y^7 + 7\,y^6x + 8\,y^5x^2 - 3\,y^4x^3 - 10\,y^3x^4 - 10\,y^2x^5 - x^7 - 33\,y^6 \\ & - 29\,y^5x - 13\,y^4x^2 + 26\,y^3x^3 + 30\,y^2x^4 + 10\,yx^5 + 3\,x^6 + 33\,y^5 \\ & + 37\,y^4x - 8\,y^3x^2 - 33\,y^2x^3 - 20\,yx^4 - 3\,x^5 - 11\,y^4 - 15\,y^3x \\ & + 13\,y^2x^2 + 10\,yx^3 + x^4, \end{aligned}$$

is the degree-7 polynomial considered in Example 1.2.4, part 3.