UNIVERSITÄT DES SAARLANDES
Fachrichtung 6.1 - Mathematik
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Exercises Algebraic Geometry

Winterterm 2016/17

The solutions are collected on Tuesday, before the exercise session.
All further informations concerning the lecture can be found here: https://www.math.uni-sb.de/ag/schreyer/index.php/teaching

Sheet 9 09.01.2017

Exercise 1. Draw the projective curves defined below in the 3 standard affine charts. Also draw the tangents in intersection points with the coordinate axes.

(a) $x^2 - \frac{1}{4}y^2 - z^2 = 0$
(b) $yz^2 - x^3 = 0$

Exercise 2. (a) Let $C_1, C_2 \subset \mathbb{P}^2(\mathbb{C})$ be two smooth conics, that is, smooth plane curves of degree 2. Prove that they are *projectively equivalent*, which means that there is an automorphism $A \in \mathrm{PGL}(3, \mathbb{C})$ such that $A(C_1) = C_2$.

(b) How many classes of smooth projective conics exist in $\mathbb{A}^2(\mathbb{R})$ respectively $\mathbb{P}^2(\mathbb{R})$ up to linear automorphism?

Exercise 3. Let $p_0, \ldots, p_n, p_{n+1} \in \mathbb{P}^n$ be a collection of $n + 2$ points such that no subset of n points lies on a hyperplane. Prove that there exists a unique automorphism $A \in \mathrm{PGL}(n + 1, \Bbbk)$ such that

$$Ap_0 = [1 : 0 : \cdots : 0], \ldots, Ap_n = [0 : \cdots : 0 : 1] \text{ and } Ap_{n+1} = [1 : \cdots : 1].$$

We frequently refer to the points $[1 : 0 : \cdots : 0], \ldots, [0 : \cdots : 0 : 1]$ as the *coordinate points* and to $[1 : \cdots : 1]$ as the scaling point.

Exercise 4. Let $p_1, p_2, p_3 \in \mathbb{R}^2$ be three non colinear points. Prove that there exists a unique circle passing through them.
Hint: Establish that the set of circles in an affine chart is a 3-dimensional linear system $L \subset \mathbb{P}(\mathbb{R}[x, y]_{\leq 2})$. The base points of this system are called the *circle points*. Where are they?