Abstracts of Talks

Commutant algebras for certain differential operators in $L^p(\mathbb{R})$

E. Albrecht

In this talk, we investigate the commutant and bicommutant algebras in $\mathcal{L}(L^p(\mathbb{R}))$, $1 \leq p < \infty$ for differential operators $Q\left(\frac{1}{i}\frac{d}{dx}\right)$, where Q is a real polynomial. Some of the effects due to the multiplicity of Q when $p = 2$ no longer occur for $p \neq 2$, except when Q is symmetric about some axis. If Q has no such symmetry, then the commutant algebra in $\mathcal{L}(L^1(\mathbb{R}))$ coincides with the algebra of all 1-multiplier operators.

Fréchet algebras and formal power series

G.R. Allan

In 1972 [1], we gave conditions on a commutative Banach algebra R, necessary and sufficient for the algebra $\mathbb{C}[[X]]$, of all complex formal power series in one variable, to be embeddable in R. This result had consequences in automatic continuity.

Much more recently [2] we determined those Fréchet algebras in which $\mathbb{C}[[X]]$ may be embedded. This later work, in some sense, completes the earlier result. It also yields new applications in automatic continuity theory, as well as some new ideas in the theory of inverse limits (discussed in [3]) which have a number of interesting applications.

In the talk, we shall explain the new ideas introduced in [2] and [3]. We shall discuss implications for automatic continuity and also some questions concerning the embedding of algebras $C(\Omega)$ in Fréchet algebras, which require concepts not needed in the Banach-algebra theory.

References

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[2] —, *Fréchet algebras and formal power series*, Studia Math. **119** (1996), 271–288.

[3] —, *Stable inverse-limit sequences, with application to Fréchet algebras*, Studia Math. **121** (1996), 277–308.

On the spectrum of the C^*-algebra of a nilpotent Lie group

R. J. Archbold

A C^*-algebra is said to be *quasi-standard* if the failure of separation by disjoint open sets defines an open equivalence relation on the primitive ideal space. In the separable case, quasi-standardness is equivalent to the algebra being representable as a continuous C^*-bundle such that the fibres are primitive throughout a dense subset of the base space.

In this talk we shall discuss recent results from [1] and [2] concerning the question of quasi-standardness for $C^*(G)$ when G is a nilpotent Lie group. A sufficient condition is given in terms of the Kirillov orbit structure in the dual of the Lie algebra. This condition is necessary in the 2-step case but is not necessary in the 3-step case. Indeed, an unexpectedly large number of six-dimensional nilpotent Lie groups turn out to be quasi-standard.

We shall also discuss quasi-standardness and separated points in $\hat{G}$ for two different infinite families of nilpotent Lie groups each of which generalizes properties of the classical Heisenberg group G_3.

References

[1] R. J. Archbold and E. Kaniuth, *Simply connected nilpotent Lie groups with quasi-standard C^*-algebras*, Proc. Amer. Math. Soc. (to appear).

[2] R. J. Archbold, E. Kaniuth, G. Schlichting and D. W. B. Somerset, *On the topology of the dual of a nilpotent Lie group*, preprint.

Spectrum-preserving mappings

B. Aupetit

The first part of this talk will be a general survey on a great number of results concerning the complex linear mappings, from a Banach algebra into another one, which preserve the spectrum. In particular, it will contain some indications on the proof of a conjecture of L.A. Harris and R.V. Kadison concerning such mappings in the case of finite von Neumann algebras. The second part will be devoted to the difficult study of the additive mappings which preserve the spectrum. In the last part some results on the multiplicative mappings which preserve the spectrum shall be given.

Splitting of finite dimensional extensions of some commutative Banach algebras

William G. Bade

This talk will describe some results from a memoir with H. G. Dales and Z. A. Lykova. Let A be a commutative unital Banach algebra. We consider extensions $\Sigma : 0 \to I \to \mathfrak{A} \to A \to 0$ of A, where $\mathfrak{A}$ is a Banach algebra and I is a closed finite dimensional ideal in A. We wish to determine when Σ splits strongly (respectively, algebraically) in the sense that there is a continuous homomorphism (respectively, a homomorphism) $\theta : A \to \mathfrak{A}$ with $\pi \circ \theta$ the identity on A, where $\pi : \mathfrak{A} \to A$ is the canonical epimorphism.

We obtain splitting results when, for each $\phi \in \Phi_A$, the maximal ideal M_ϕ is either principal or contains a closed primary ideal of finite codimension which has an approximate identity of some sort. The theorems are applicable to the algebras $C^n(\mathbb{I})$ of n-times continuously differentiable functions on the unit interval and to the Beurling algebras A_α^+, $\alpha > 0$, of analytic functions f on the unit disk for which $\sum_{n=0}^{\infty} |\hat{f}(n)|(1+n)^\alpha < \infty$. Besides obtaining positive results on splitting, we describe extensions which do not split even algebraically. Various open problems are discussed.

References

[1] W. G. Bade, H. G. Dales, and Z. A. Lykova, *Algebraic and strong splittings of extensions of Banach algebras*, submitted.

Functions that operate on Banach algebras and Banach spaces of continuous functions vanishing at infinity

E. Briem

One formulation of the Stone-Weierstrass Theorem is the following: If the function $h(t) = t^2$ operates by composition on a uniformly closed space B of continuous real-valued functions on a compact Hausdorff space X, where B separates the points of X and contains the constant functions, then $B = C(X)$. Clearly we do not have the same characterization of $C(X)$ among Banach function spaces on X i.e. spaces complete in some norm which dominates the sup-norm. It is, however, possible to characterize $C(X)$ among Banach function algebras or even Banach function spaces on a compact Hausdorff space X by means of operating functions. In [1], Y. Katznelson proved that if the $h(t) = \sqrt{t}$ operates on a real Banach function algebra of continuous functions on a compact Hausdorff space X, then that algebra must be all of $C(X)$. Later in [2] O. Hatori generalized this result to Banach function spaces.

In our talk we will discuss results along similar lines for Banach function algebras and Banach function spaces of continuous functions vanishing at infinity on locally compact spaces. We mention the following result, a slight generalization of a theorem by Katznelson: If B, a Banach subalgebra or subspace of c_0, has an operating function h defined on $[0, 1)$ with the property that $h(t)/t \to \infty$ as $t \to 0^+$, then $B = c_0$.

References

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[2] O. Hatori, *Symbolic calculus on a Banach algebra of continuous functions*, J. Funct. Anal. **115** (1993), 247-280.

Spectral continuity for adjoint operators

Laura Burlando

For every complex Banach algebra A, let $\sigma_A\colon A \to \mathbb{K}_{\mathbb{C}}$ (where $\mathbb{K}_{\mathbb{C}}$ denotes the set of all compact nonempty subsets of the complex plane $\mathbb{C}$, endowed with the Hausdorff metric) denote the spectrum function on A.

This talk will be mainly concerned with the relationship between continuity of $\sigma_{L(X)}$ (where $L(X)$ denotes the Banach algebra of all bounded linear operators on a complex Banach space X) at $T \in L(X)$ and continuity of $\sigma_{L(X^*)}$ at the adjoint T^* of T. In particular, we will show that, if $\sigma_{L(X)}$ is continuous at T, $\sigma_{L(X^*)}$ need not be continuous at T^*.

Localized L^1-factorization for absolutely continuous contractions

I. Chalendar

Let $\mathcal{H}$ be a separable, infinite-dimensional complex Hilbert space and let $\mathcal{L}(\mathcal{H})$ denote the algebra of all bounded linear operators on $\mathcal{H}$. If $T \in \mathcal{L}(\mathcal{H})$ is an absolutely continuous contraction, then, for any $x, y \in \mathcal{H}$, there exists a function $x{\cdot}y \in L^1$ such that the Fourier coefficients of $x{\cdot}y$ satisfy: $(x{\cdot}y)^\wedge(n) = (T^{*n}x, y)$ if $n \geq 0$ and $(x{\cdot}y)^\wedge(n) = (T^{-n}x, y)$ if $n < 0$. (see Proposition 8.3 in [2]). In 1988, H. Bercovici and B. Chevreau proved independently that if $T \in \mathcal{L}(\mathcal{H})$ has an isometric Sz.-Nagy-Foias functional calculus, then, for any function $f \in L^1$, there exist $x, y \in \mathcal{H}$ such that $f^\wedge(-n) = (T^n x, y)$, $n \geq 0$ (see [1,5]).

In this paper, we study the possibility of solving exactly equations of the form $f = x{\cdot}y$ (that is, for any $n \in \mathbb{Z}$, $(x{\cdot}y)^\wedge(n) = f^\wedge(n)$) where f is a given function in L^1 and where $T \in \mathcal{L}(\mathcal{H})$ is an absolutely continuous contraction.

In the case where $T = S$, where S is the usual shift on H^2 (so that $x{\cdot}y = x\overline{y}$ for $x, y \in H^2$), this question has been solved by J. Bourgain in 1986 ([3]): the above factorization holds for $f \in L^1, f \neq 0$ if and only if $\log|f| \in L^1$. We show that for any $f \in L^1(\mathbb{T})$, there exist $x, y \in \mathcal{H}$ such that $f_{|X_T} = (x{\cdot}y)_{|X_T}$ where X_T is the Borel subset of $\mathbb{T}$ introduced in [4] and which coincides with $\mathbb{T}$ if and if $T \in \mathbb{A}_{\aleph_0}$ (e.g. the *Bergman shift* is in this class). Thus, in particular, we obtain a new characterization of the class $\mathbb{A}_{\aleph_0}$. Finally, we try to discuss factorizations of the form $f = x{\cdot}y$ where f is a non-trivial log-integrable function in L^1 and where T is an absolutely continuous contraction whose residual part or $*$-residual part has a spectral measure supported by the unit circle. For this purpose we combine a result of Kérchy about injections of unilateral shifts into contractions (cf. [6]) and a result of Bourgain about a problem of Douglas and Rudin (see [3]).

References

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[2] H. Bercovici, C. Foias, and C. Pearcy, *Dual algebras with applications to invariant subspaces and dilation theory*, CBMS Regional conference series in mathematics **56** (1985).

[3] J. Bourgain, *A problem of Douglas and Rudin of factorization*, Pac.J. Math. **121** (1986), 47–50.

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Membership in the class $\mathbb{A}_{\aleph_0}$ for commuting N-tuples of contractions

B. Chevreau

The following theorem is a standard application of the initial S. Brown method to (Hilbert space) contractions.

Theorem. (Apostol-Bercovici-Foias-Langsam-Pearcy-Robel 82-84)
Let T be an absolutely continuous contraction whose essential spectrum is dominating in the open unit disc $\mathbb{D}$. Then:

(1) The approximation set $\mathcal{X}_0(T)$ is equal to the closed unit ball the predual $\mathcal{Q}$ of $H^\infty(\mathbb{D})$,
(2) $T \in \mathbb{A}_{\aleph_0}$ and
(3) T is reflexive.

Several partial generalizations of this theorem to the context of commuting N-tuples with a unitary dilation have been obtained. Up to last year they included additional hypothesis (e.g. the N-tuple is of type $C_{0\cdot}$ and/or $C_{\cdot 0}$, the N-tuple is doubly commuting and/or restrictions on the nature of the spectrum). The purpose of the talk is to sketch the recent proof (by the author) of the analogue of this theorem (with the notion of Taylor essential spectrum) for absolutely continuous N-tuples of contractions admitting a unitary dilation.

Second order cohomology groups for Banach algebras

H. G. Dales

I shall first recall the definitions of the second cohomology groups of a Banach algebra, and discuss their role in the theory of decompositions of a Banach algebra. I shall then describe some calculations of specific second cohomology groups; the algebras will be often be the semigroup algebras A of certain discrete semigroups, and the coefficient bimodules for the cohomology groups will often be $A\hat{\otimes}A$ and its dual module. This material is taken from joint work with J. Duncan (University of Arkansas).

Compact elements in discrete convolution semigroup algebras

H.V. Dedania

Let S be a unital, cancellative, abelian, additive semigroup, and let ω be a weight on S; i.e., $0 < \omega(s+t) \leq \omega(s)\omega(t) \quad (s,t \in S)$. Then the Banach space $l^1(S,\omega)$, the set of all complex-valued functions f on S such that $f\omega \in l^1(S)$, is a unital, commutative Banach algebra with respect to the convolution product. Let A be a commutative Banach algebra, and let $a \in A$. Then a is called a *compact element* in A if the operator T_a, where $T_a(b) = ab \quad (b \in A)$, is compact in the usual sense.

Here we prove the following results: Let $d > 0$. Then there exists a weight ω on Q^+ (depending on d) such that the set of all compact elements in $l^1(Q^+,\omega)$ is exactly the standard closed ideal $M_{(d)}$ or $M_{[d]}$. Moreover, there does exist a weight ω on Q^+ such that every element $f \in l^1(Q^+,\omega)$, where $f(0) = 0$, is compact.

Hochschild cohomology groups of certain algebras of analytic functions with coefficients in one-dimensional bimodules

O. Ermert

We compute the algebraic and continuous Hochschild cohomology groups of certain Fréchet algebras of analytic functions on a domain U in $\mathbb{C}^n$ with coefficients in one-dimensional bimodules. Among the algebras considered, we focus on $A = A(U)$. For this latter algebra, our results apply if U is smoothly bounded and strictly pseudoconvex, or if U is a product domain.

Reflexivity of subnormal tuples with rich spectrum in the ball or the polydisc

J. Eschmeier

In 1978 Scott Brown proved that each subnormal operator on a complex Hilbert space has a non-trivial invariant subspace. Using Scott Brown's methods Olin and Thomson showed in 1980 that each subnormal operator is reflexive, and thus extended results of Sarason on the reflexivity of normal operators and analytic Toeplitz operators and of Deddens on the reflexivity of isometries. It was shown by K. Yan that each subnormal n-tuple on a complex Hilbert space has a non-trivial joint invariant subspace. Apart from a result of Bercovici saying that all commuting families of isometries are reflexive no general reflexivity result for subnormal tuples seems to be known. In the planned talk we want to indicate that all subnormal tuples with rich spectrum in the unit ball or the unit polydisc are reflexive. At the same time we obtain invariant-subspace results for spherical contractions and commuting tuples of Hilbert-space contractions with sufficiently rich spectrum.

Translation biinvariant subspaces for weighted Hilbert spaces on $\mathbb{Z}$

J. Esterle

We study translation invariant subspaces for weighted Hilbert spaces on $\mathbb{Z}$. In all the situations considered the weight w is nonincreasing, the spectrum of the shift operator is the unit circle, the weight w satisfies the usual quasianalyticity condition on the negative integers and the product $w(n)w(-n)$ tends to infinity as n tends to infinity. Our main result shows that if the product $w(n)w(-n)$ grows fast enough, and if w satisfies suitable regularity conditions then all closed translation biinvariant subspaces M are generated by their "analytic" part, i.e. by the set of elements of M for which the negative coefficients vanish. In the case where $w(n) = 1$ for n positive this means that all closed translation biinvariant subspaces are generated by the Fourier transform of some singular inner function. In the general case proving the existence of nontrival closed translation biinvariant subspaces leads to open problems concerning z-invariant subspaces for weighted Hardy spaces satisfying some division conditions. Deep positive results in this direction were recently obtained by A. Atzmon using the theory of entire functions, and A. Borichev solved positively the related question of existence of zero free z-invariant subspaces.

The methods are based on an extensive use of the Borichev-Volberg theory of asymptotically holomorphic functions in the disc, the main results were obtained in collaboration with A. Volberg.

Strong regularity and characters for Banach function algebras

J. F. Feinstein

In our talk we shall discuss the condition of strong regularity for Banach function algebras (especially uniform algebras), and related questions about the character space. It is standard that any finite set of distinct characters on a commutative Banach algebra is linearly independent. We shall present an elementary result of Wolff from 1921 (see [1]) which, when interpreted in terms of Banach algebras, shows that it is possible to find a sequence of distinct characters ϕ_n on the disk algebra and an l_1 sequence of complex numbers λ_n, not all zero, such that $\sum_{n=1}^{\infty} \lambda_n \phi_n = 0$. However, we shall see that, even for general commutative Banach algebras, this is not possible if the countable sequence of characters forms a compact set. Using this latter result we can prove the following theorem, which strengthens earlier results of Wilken and the author.

Theorem. Let A be a uniform algebra on a compact space X. Suppose that A is strongly regular at all but at most countably many points of X. Then the character space of A is precisely X.

We shall also discuss some related examples, results and questions.

References

[1] J. Wolff, *Sur les séries* $\sum \frac{A_k}{z-\alpha_k}$, C. R. Acad. Sci Paris **173** (1921), 1057–1058, 1327–1328.

Q-algebras: how close they are to Banach algebras

Maria Fragoulopoulou

A topological algebra is called a Q-algebra if the set of its quasi-invertible elements is open. This concept was introduced by I. Kaplansky in early 40's. Every Banach algebra is a Q-algebra, but there are important examples of non-normed topological algebras which are Q-algebras. For instance, the algebra of all smooth functions on the closed unit interval as well as the algebra of all smooth functions on the real line, with compact support. Among non-complete normed algebras there are also interesting examples of Q-algebras. In our talk we shall discuss several properties of Q-algebras which make them quite similar to Banach algebras and besides we shall point out nice properties possessed by some Q-algebras, but not by Banach algebras.

Virtual diagonals and averaging

José E. Galé

It is shown that averaging with virtual diagonals in amenable Banach algebras provides a sort of unified way to deal with some aspects of the cohomology of Banach algebras. In particular this enables us to simplify and extend results on geometry of spaces of representations obtained in previous papers.

References

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The "triple" norm extension problem

A. Moreno Galindo

As is the case of Jordan algebras, in the normed treatment of the Zel'manov classification on Jordan Triple Systems (JTS) it would be interesting to know when a norm in a special JTS is the restriction of an algebra norm on a reasonable algebra envelope. We give an intrinsic and general criterion for this question:

Theorem. Assume (A, τ, π) is an associative algebra over $K(=\mathbb{R}, \mathbb{C})$, with two commuting involutions, which is a τ-π-tight envelope of $T := H(A,\tau) \cap S(A,\pi)$, and let $|\cdot|$ be a norm on the JTS T. Then the following assertions are equivalent:

(1) There exists an algebra norm $\|\cdot\|$ on A making τ and π isometric and whose restriction to T is equivalent to $|\cdot|$.
(2) The pentad mapping $(t_1, t_2, t_3, t_4, t_5) \mapsto \frac{1}{2}(t_1t_2t_3t_4t_5 + t_5t_4t_3t_2t_1)$ from $T \times T \times T \times T \times T$ into T is $|\cdot|$-continuous.

Moreover, we obtain an analytical characterization of the elemets of the free special JTS: There exists a norm $\|\cdot\|$ on the JTS $M_\infty(K)$ such that the triple-polynomials are precisely those associative polynomials wich act $\|\cdot\|$-continuously on $(M_\infty(K), \|\cdot\|)$.

Derivations into iterated duals of Banach algebras

Fereidoun Ghahramani

The notion of weak amenability was introduced by Bade, Dales and Curtis for commutative Banach algebras and by Johnson for any Banach algebra. A Banach algebra A is weakly amenable if $\mathcal{H}^1(A, A^*) = \{0\}$. We (H. G. Dales, F. Ghahramani, and N. Grønbæk) are recently studying the question of when $\mathcal{H}^1(A, A^{(n)}) = \{0\}$, where $A^{(n)}$ is the n-th dual space of A, $n = 1, 2, \ldots$. We call a Banach algebra A n-weakly amenable if $\mathcal{H}^1(A, A^{(n)}) = \{0\}$, and permanently weakly amenable, if A is n-weakly amenable for all n.

In this talk, I will first review some of the relations between m-weak amenability and n-weak amenability. Then I will sketch a proof of a result that for any locally compact group G, the group algebra $L^1(G)$ is $(2n+1)$-weakly amenable. I will also show that the algebra $\mathcal{N}(E)$ of nuclear operators on an infinite dimensional Banach space E is weakly amenable but not 2-weakly amenable, provided E^* has the approximation property. Every C^*-algebra is permanently weakly amenable.

Operator algebras with a reduction property

J. A. Gifford

We consider the class of operator algebras for which $\mathcal{H}^1(A, B(H)) = 0$ whenever $A \to B(H)$ is a bounded representation. Amongst the C^*–algebras this property — which we call the total reduction property — characterises those algebras for which every representation is similar to a $*$-representation.

A non-self-adjoint algebra with the total reduction property has many 'C^*-algebra like' qualities. We show that if $A \subseteq K(H)$ is an operator algebra of compact operators with this property then A is isomorphic to a C^*-algebra.

The proof of this result is facilitated by Lomonosov's partial solution to the transitive algebra problem for the special case of algebras containing compact operators [1]. An understanding of the structure of more general algebras with the total reduction property appears to require a better understanding of the transitive algebra problem.

References

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Generalizations of Fredholm Operators

Sandy Grabiner

Let T be a bounded operator on a Banach space. For each $n \geq 0$, T induces a map from $R(T^n)/R(T^{n+1})$ onto $R(T^{n+1})/R(T^{n+2})$ as well as an injective map from $N(T^{n+2})/N(T^{n+1})$ into $N(T^{n+1})/N(T^n)$. We observed some years ago that, if either of these induced maps is an isomorphism, then both are. More generally, they fail to be isomorphisms by the same amount; that is, if we let $k_n(T)$ be the dimension of the null-space of the first of the induced maps, then $k_n(T)$ is also the co-dimension of the range of the second induced map. We will start by reviewing and extending our earlier results for maps T with $k_n(T) = 0$ (that is, when the induced maps are isomorphisms) for sufficiently large n, and then consider the more general case when there is a $d \geq 0$ for which $k_n(T)$ is finite for all $n \geq d$. Notice in particular that $k_n(T) = 0$ for sufficiently large n if T is semi-Fredholm or has finite ascent or descent.

In order to go beyond purely algebraic results, we need to assume a topological condition on $R(T^n)$ for some $n > d$ (which turns out to imply the same condition for all $n > d$). Our condition is automatically satisfied if $R(T^n)$ is closed but is also satisfied if $R(T^n)$ has finite co-dimension in $R(T^{n-1})$. We will describe the relationship of our topological assumption to $R(T^n)$ actually being closed. For the maps T we consider, we also study maps induced on subspaces and quotient spaces, and we study perturbations of T.

Fréchet operator algebras with spectral invariance and the evolution of singularities

B. Gramsch

In the microlocal analysis of partial differential operators it turned out that it is useful to consider special topological algebras. Let us assume for a unital, symmetric subalgebra Ψ of a C^*-algebra B with unit the following properties (B^{-1} and Ψ^{-1} denote the respective groups of invertible elements):

(1) $\Psi \cap B^{-1} = \Psi^{-1}$ (spectral invariance, inverse closed);
(2) Ψ is a submultiplicative locally convex Fréchet algebra continuously embedded into B.

In many contributions, e.g. of Bony, Chemin, Cordes, Schrohe and Lauter, it has been shown that the above properties are fulfilled for important classes of operators in the pseudodifferential analysis. In this talk we apply a localization technique for the above Ψ^*-algebras to prove a version of the Weyl-Lemma (elliptic regularity) with operator theoretic methods such that also in special situations new results are produced for classical pseudodifferential operators. The Hörmander-Sato result on the propagation of singularities for solutions of operators of real principle type is generalized to the Ψ^*-setting. We define a phase space also in the case when the quotient algebra modulo compact operators is non commutative. Then it can be shown that the non commutative wave front of certain operators propagates along generalized bicharacteristics. This is applied to systems of scalar operators where the general phase space can be computed. The approach depends also on a theory of Green inverses, relative inverses and semi Fredholm operators in the above Fréchet operator algebras.

References

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Some concepts from group cohomology in the Banach algebra context

N. Grønbæk

An obvious approach to the computation of the cohomology of a group is to try to compare it to the cohomology of its subgroups. Algebraic tools for this are Shapiro's Lemma and transfer maps. In the talk we shall describe a Banach algebraic framework that makes these concepts applicable to convolution group algebras. Immediate results are statements like: The weak homological bidimensions of the group algebras for $GL(n,\mathbb{R})$, $SL(n,\mathbb{R})$ and $SL(n,\mathbb{Z})$ coincide.

The generalized Carleson condition in certain spaces of analytic functions

A. Hartmann

A classical result of L. Carleson describes the sequences Λ in the unit disk such that every bounded sequence is interpolable on Λ by a bounded analytic function in the disk. This notion of interplation may be generalized in the following way. Let X be a space of analytic functions on the unit disk. We say that a function $f \in X$ interpolates a sequence $(f_n)_{n\geq 1} \subset X$ with respect to a sequence of inner function $(\vartheta_n)_{n\geq 1}$, if for all $n \geq 1$ we have $f - f_n \in \vartheta_n X$, (cf. [2]). Using Hilbert space techniques and the fact that the space of bounded analytic functions on the unit disc H^∞ is the multiplier algebra of H^2, V. I. Vasyunin (cf. [3]) had proved that the so-called generalized Carleson condition is necessary and sufficient for free interpolation of sequences of constant functions $f_n \equiv a_n$ with respect to the sequence $(\vartheta_n)_{n\geq 1}$. This result was generalized to uniformely bounded sequences of H^∞-functions by N. K. Nikolski [2]. In [1] we adapted Vasyunin's (and Nikolski's) reasoning to show that this result remains true in Hardy spaces H^p, $1 < p < \infty$.

The aim of this talk will be to prove that this result may be extended to a large class of holomorphic functions on the unit disc. The key to this extension is a generaliztion of the lifting commutant theorem based on Nehari's theorem.

References

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Projective homological classification of C^*-algebras

A. Ya. Helemskii

What can be said about the structure of a given C^*-algebra if we have an information about the size of its stock of projective Hilbert modules (in what follows, PHM)? We characterize algebras that have at least one non-trivial PHM, and algebras that have sufficient, to distinguish their elements, family of PHM. (We prove, in particular, that unital simple C^*-algebras do not have any non-trivial PHM at all). Apart from this, we show that the class of PHM over a C^*-algebra contains all irreducible modules iff the algebra is the restricted sum of a family of algebras of compact operators, and contains all non-degenerate modules iff this algebra is the restricted sum of a family of full matrix C^*-algebras.

Preparing the proofs of the mentioned theorems, we give a complete description (up to an isometric isomorphism) of all PHM over a given C^*-algebra.

Symmetric amenability

B. E. Johnson

A Banach algebra A is amenable if it has a bounded approximate diagonal, in fact this can be used as the definition of amenability. In the tensor algebra $A\hat{\otimes}A$ we have the flip map $(a \otimes b)^\circ = b \otimes a$ and the symmetric tensors are those invariant under the flip map. A Banach algebra is *symmetrically amenable* if it has an approximate diagonal consisting of symmetric tensors. Most amenable Banach algebras are symmetrically amenable, in fact the two concepts are the same for commutative algebras and group algebras but there are amenable Banach algebras which are not symmetrically amenable. Symmetric amenability can be used to resolve questions about Lie and Jordan derivations.

References

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Endomorphisms of Banach algebras of infinitely differentiable functions

Herbert Kamowitz

Let M_n denote a sequence of positive numbers satisfying $M_0 = 1$, and $\binom{m+n}{n} \le \frac{M_{m+n}}{M_m M_n}$, and let

$$D(M_n) = \left\{ f \in C^\infty[0,1] : \|f\|_{D(M_n)} = \sum_{n=0}^{\infty} \frac{\|f^{(n)}\|_\infty}{M_n} < \infty \right\}.$$

If $\lim_{n\to\infty} \left(\frac{n!}{M_n}\right)^{1/n} = 0$, then the maximal ideal space of $D(M_n)$ is precisely $[0,1]$, and every nonzero endomorphism T of $D(M_n)$ has the form $Tf = f \circ \phi$ for some $\phi \in D(M_n)$, $\phi : [0,1] \to [0,1]$. We show that under mild restrictions, a necessary condition that ϕ induces an endomorphism of

$D(M_n)$ is that $\|\phi'\|_\infty \le 1$, and for some of these algebras this condition is also sufficient. In any case, with mild restrictions, if $\lim_{n\to\infty}\left(\dfrac{n!}{M_n}\right)^{1/n} = 0$, then $\|\phi'\|_\infty < 1$ is sufficient for ϕ to induce an endomorphism of $D(M_n)$. Moreover, if $\|\phi'\|_\infty < 1$, then the endomorphism induced by ϕ is compact.

On the dual Banach algebra of bounded uniformly continuous functions

A. Lau

Let G be a locally compact group, and let $LUC(G)$ be the C^*-algebra of bounded left uniformly continuous function on G. Then G may be regarded as a subset of the dual Banach space $LUC(G)^*$. The multiplication on G may be extended to a multiplication on $LUC(G)^*$ such that $LUC(G)^*$ is a Banach algebra and the spectrum of $LUC(G)$, (KG, weak^*), with this product is a compact semigroup. In this talk, I shall explore some recent results concerning the Banach algebra $LUC(G)^*$ and the compact semigroup KG.

Spectra and essential spectra of multipliers connected via similarities

Kjeld B. Laursen

If two operators are 'weakly' similar ('weakly' is a non-technical term with several possible meanings, all to be explained in this talk), their spectra and essential spectra will overlap. With a start in the general local spectral theory we shall describe ways of identifying the Fredholm regions of 'weakly' similar operators, thus also obtaining information about essential spectra. These results will be applied to multipliers on commutative semi-simple Banach algebras, in particular to convolution operators on group algebras.

Weak Amenability of C^*-Algebras and a Theorem of Goldstein

N. J. Laustsen (joint work with U. Haagerup)

A Banach algebra A is said to be weakly amenable provided that every bounded derivation from A into its dual A^* is inner. In [3], U. Haagerup proved that every C^*-algebra is weakly amenable, building on earlier work by J. W. Bunce and W. L. Paschke [1]. We give a simplified and unified proof of this result. Moreover, we prove that every bounded Jordan derivation from a C^*-algebra A into its dual A^* is a derivation and consequently inner.

As an application of this result we give an elementary proof of the following theorem of S. Goldstein [2]:

Theorem. Let A be a C^*-algebra, and let $V : A \times A \to \mathbb{C}$ be a bounded bilinear form. Then the following assertions are equivalent:

(a) $\forall a, b \in A_{\mathrm{sa}} : ab = 0 \Longrightarrow V(a,b) = 0$;

(b) $\exists \varphi, \psi \in A^* \; \forall a, b \in A : V(a,b) = \varphi(ab) + \psi(ba)$.

Moreover, the functionals in (b) can be chosen to be positive if and only if $V(c, c^*) \ge 0$ for all $c \in A$.

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The Structure of Ψ^*-algebras of totally characteristic operators on manifolds with boundary

Robert Lauter

Ψ^*-algebras, i.e. symmetric, spectrally invariant Fréchet subalgebras of C^*-algebras, were introduced by Gramsch in 1981 in connection with a perturbation theory for Fréchet-algebras. Since then many algebras occuring naturally in the pseudodifferential analysis were shown to be Ψ^*-algebras. However, the algebra $\Psi^0_{b,cl}(X, {}^b\Omega^{\frac{1}{2}})$ of b-pseudodifferential operators on a compact manifold X with boundary introduced by Melrose in 1981 turns out to be not a Ψ^*-algebra; to circumvent this problem we constructed a Ψ^*-algebra $\mathcal{A}(X)$ containing the algebra $\Psi^0_{b,cl}(X, {}^b\Omega^{\frac{1}{2}})$ as a dense subalgebra, reflecting the $\mathcal{C}^\infty$-structure of X, and having the following additional properties:

(1) There is a symbol map $\tau : \mathcal{A}(X) \longrightarrow Q_\infty \subseteq Q$ characterizing the Fredholm property of $a \in \mathcal{A}(X)$ by means of the invertibility of $\tau(a)$; here Q_∞ is a Ψ^*-algebra of "smooth" symbols.
(2) $\mathcal{A}(X) \subseteq \bigcap \mathcal{L}(H^s)$ where $H^s = H^s(X, {}^b\Omega^{\frac{1}{2}})$ is the scale of b-Sobolev spaces.
(3) Separate from the boundary one recovers the ordinary pseudodifferential calculus.

Moreover, the C^*-closure $\mathcal{B}(X)$ of $\mathcal{A}(X)$ is shown to be solvable of length 2, the relationship between certain closed ideals is clarified, spectrum and Jacobson topology of $\mathcal{B}(X)$ resp. $\mathcal{A}(X)$ are computed explicitly in terms of the underlying manifold X; in fact, the spectra of $\mathcal{B}(X)$ and $\mathcal{A}(X)$ turned out to be homeomorphic. Parts of the results are related to the work of Mantlik, Melrose, Nistor, Plamenevskij and Schulze.

Amenable Banach algebras with discrete spectrum

Richard J. Loy

The spectrum of an amenable commutative Banach algebra A is known to be discrete under the norm topology [1]; uniformly discrete under the norm topology [2]; discrete under the weak topology [3]. Here we are interested in amenable algebras whose spectrum is discrete under the weak* topology; such will be the case, for example, whenever A is an ideal in A^{**} [3].

We consider various results for such algebras, including their possible relation to the commutative amenable radical algebra problem, and indicate some of the open questions in the area.

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Computing Banach simplicial and cyclic (co)homology groups

Z. A. Lykova

Significant results of A. Connes on non-commutative differential geometry [1] have led much research interest to the computation of cyclic (co)homology groups in recent years. A promising approach to the calculation of cyclic cohomology groups is to break it down by making use of extensions of Banach algebras. We say that the extension has *the excision property in a particular homology* if there exists an associated long exact sequence of the corresponding homology groups.

We prove that an extension of Banach algebras $0 \to B \to A \to D \to 0$, such that B has a left or right bounded approximate identity, has the excision property in Banach simplicial or cyclic cohomology groups if and only if it has one for homology groups. We also present the continuous version of a result of Wodzicki (see [2]) on the existence of associated long exact sequences of simplicial and cyclic homology groups. From these two results we deduce the following thorem.

Theorem. For every extension of Banach algebras $0 \to B \to A \to D \to 0$ such that B has a left or right bounded approximate identity, there exist associated long exact sequences of Banach simplicial and cyclic (co)homology groups.

In particular, such long exact sequences exist for every extension of C^*-algebras.

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Inner derivations on ultraprime Banach algebras

J. Martinez Moreno

This is a joint work with M. Cabrera. We show that the set of all inner derivations of an ultraprime real Banach algebra is closed within all bounded derivations. More concretely, we show that for such an algebra A there exists a positive number γ (depending only on the "constant of ultrapriminess" of A) satisfying $\gamma\|a + Z(A)\| \leq \|D_a\|$ for all a in A, where $Z(A)$ denotes the centre of A and D_a denotes the inner derivation on A induces by a. The proof relies on the following theorem: ultraproducts of a family of central ultraprime real Banach algebras with a unit and with constant of ultrapriminess greater that or equal to a fixed positive constant are central ultraprime Banach algebras with a unit. This fact is obtained via a general result for real Banach algebras that reads as follows: If A is a central real Banach algebras with a unit $\mathbf{1}$, then for every a in A satisfying $\|\mathbf{1} + a^2\| < 1$ we have

$$\left[1 + \sqrt{1 - \|\mathbf{1} + a^2\|}\right]^2 \leq 2(\|I + M_a\| + \|D_a\|),$$

where M_a denotes the two-sided multiplication operator by a on A.

Alling's conjecture on closed prime ideals in H^∞

R. Mortini

It will be shown that every nonzero closed prime ideal in the algebra of bounded analytic functions on the open unit disk is an intersection of maximal ideals, hence is uniquely determined by its hull. Our proof is based on Carleson's original methods in proving the Corona Theorem, its extensions due to Sarason and Bourgain and on Suarez's recent results on separating properties of H^∞. The results of this talk are joint work with Pamela Gorkin [1].

References

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On the semi-Browder spectrum

V. Müller

Natural spectrum, natural local spectra, and spectral mapping theorems for multipliers on commutative Banach algebras

Michael M. Neumann

Following Zafran, a Borel measure on a locally compact abelian group is said to have natural spectrum if its spectrum is the closure of the range of its Fourier-Stieltjes transform on the dual group. In the first part of this talk, the notion of natural spectrum will be related to the local spectral theory of convolution operators on group algebras. In particular, the theory of Helson sets will be used to answer several natural questions in this context. This continues the work of Albrecht, Eschmeier, and Zafran in this direction. In the second part of this talk, we shall consider the more general case of representations of regular commutative Banach algebras and their multiplier algebras. Here we present several spectral mapping theorems for the Arveson spectrum together with their local counterparts. The talk will conclude with some applications to the theory of group representations.

Closed algebras of smooth functions

Anthony G. O'Farrell

There is as yet no satisfactory description of the closed subalgebras of the algebra of infinitely-differentiable real-valued functions on a smooth manifold. The same is true of the algebra of C^k functions, for $k \geq 1$.

The Stone-Weierstrass Theorem solves the problem for C^0 functions. Whitney's Spectral Theorem provides a description of the closed *ideals* in the general case. Nachbin described the *maximal* closed subalgebras of C^k in 1949, and he responded to a question of Segal by proposing a conjecture about the general case.

We shall describe some further progress on the problem, and a refinement of Nachbin's conjecture. This is joint work with G. Allan, G. Kakiko and R.O. Watson.

Algebraic properties of $*$-algebras

Theodore W. Palmer

This talk will consider properties of abstract $*$-algebras which are important in functional anlysis. The material is taken from my still unpublished second volume [1].

Some conditions, true for all Banach $*$-algebras, force the $*$-algebra to be a BG^*-algebra. As an example, suppose that for every hermitian element h in a $*$-algebra A there is a non-negative real number $t \in \mathbf{R}_+$ such that $h+t$ belongs to the positive cone A^1_+ of the unitization A^1 of A. Then each element $a \in A$ satisfies

$$\inf\{t \in \mathbf{R}_+ : t^2 - a^*a \in A^1_+\} = \sup\{\|T_a\| : T \text{ a pre-}*\text{-representation on some pre-Hilbert space}\}$$

where $\|\cdot\|$ is the operator norm. This shows that A is a BG^*-algebra with Gelfand–Naĭmark seminorm given by the displayed expressions.

The relationship between other properties which hold for some, but not all Banach *-algebras will also be discussed.

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Some properties of the Pisier algebra and related spaces of functions

T.V. Pedersen

The Pisier algebra ([2]) consists of those continuous functions f on the unit circle for which

$$\sum_{n=-\infty}^{\infty} \pm \widehat{f}(n)e^{in\theta}$$

is continuous on the unit circle almost surely, that is, for almost every choice of a sequence $(\pm 1, \pm 1, \ldots)$. Our main result is that spectral synthesis holds in the Pisier algebra. In the talk, we present an outline of the proof and discuss the probabilistic methods which are needed. Of these, the most important is the Dudley-Fernique theorem on the almost sure continuity of Gaussian processes. We also present results about the ideal structure in certain algebras of analytic functions (generalizing the results in [1]). In particular, our results apply to the subalgebra of the Pisier algebra consisting of the functions that are analytic in the open unit disc. Finally, we consider similar spaces of functions that almost surely satisfies a Lipschitz condition.

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Dilations and invariant subspaces of commuting tuples

Sandra Pott

The dilation theory of Nagy and Foiaş for contractions on a Hilbert space has been used heavily to obtain nontrivial invariant subspaces for bounded linear operators with sufficiently rich spectrum, see e.g. [2].

Recently, J. Eschmeier has shown that a commuting n-tuple of operators with a spherical dilation and dominating Harte spectrum in the unit ball in $\mathbb{C}^n$ has a non-trivial joint invariant subspace. Criteria for the existence of spherical dilations have been given by Athavale, Müller and Vasilescu in terms of positivity conditions, generalizing the contraction condition $1 - T^*T > 0$ for $n = 1$ (see [1], [4]).

Let P be a complex polynomial in n variables with nonnegative coefficients and constant term 0, which linear terms are all nonzero. For P, we define the domain $\mathcal{P} = \{z \in \mathbb{C}^n : P(|z_1|^2, \ldots, |z_n|^2) < 1\}$. A commuting tuple $T = (T_1, \ldots, T_n)$ satisfying certain positivity conditions associated to P, which generalize the conditions of Athavale, Müller and Vasilescu, has its Taylor spectrum $\sigma(T)$ contained in the closure of $\mathcal{P}$. Moreover, up to similarity it has a normal dilation N, and $\sigma(N)$ is contained in the topological boundary $\partial\mathcal{P}$ of $\mathcal{P}$ (see [5]). Applying the techniques developed in [3] to the domain $\mathcal{P}$, we can show that every tuple possessing a normal dilation with Taylor spectrum in $\partial\mathcal{P}$ and dominating Harte spectrum in $\mathcal{P}$ has a nontrivial joint invariant subspace.

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Homology and Cohomology Groups of Commutative Banach Algebras and Analytic Polydiscs

L. I. Pugach

In previous papers [1], [2] the speaker has shown how one may deduce the existence of analytic structure in a neighbourhood of a maximal ideal, M, in the spectrum of a commutative Banach algebra, A, from homological assumptions. In this more recent work (with M. C. White) we again deduce the existence of analytic structure in the spectrum. However, in the present work we assume only properties of certain of the cohomology groups $H^n(A, A/M)$, rather than the stronger conditions on the homological dimension of the maximal ideal. The conclusion is correspondingly weaker: in the previous work one deduces the existence of a Gel'fand neighbourhood with analytic structure, here we deduce only the existence of a metric neighbourhood with analytic structure. The main method is to consider products of certain co-cycles to deduce facts about the symmetric second cohomology, which is known to be related to the deformation theory of algebras.

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The structure of contractible and amenable Banach algebras

V. Runde

A Banach algebra is said to be contractible if its first order cohomology with coefficients in an arbitrary Banach module vanishes; it is called amenable if this is true for its first order cohomology with coefficients in a dual Banach module. Despite the formal similarities between these two definitions, the classes of contractible and amenable Banach algebras have little in common. The only known examples of contractible Banach algebras are finite direct sums of full matrix algebras; on the other hand, there are abundantly many interesting examples of amenable Banach algebras. First, we give a survey of partial results — some of them with new proofs — which assert that a contractible Banach algebra satisfying some additional, fairly mild hypotheses in terms of Banach space geometry is a finite direct sum of full matrix algebras. Secondly, we discuss the related problem of whether every reflexive, amenable Banach algebra is a a finite direct sum of full matrix algebras. Finally, relying on results due to J. Boidol, H. Leptin and D. Poguntke on the failure of spectral synthesis in non-abelian harmonic analysis, we give an example of an amenable, radical Banach algebra thus answering an open question raised by P. C. Curtis.

Lifting the difference spectrum

David Salinger

We consider the difference spectrum of the largest and smallest closed ideals in the Wiener algebra $A(G)$ of a compact abelian group G.

If H is a closed subgroup of G and $\pi : G \to G/H$ is the canonical projection, then $E \subset G/H$ is a set of non-synthesis for $A(G/H)$ if and only if $\pi^{-1}(E)$ is a set of non-synthesis for $A(G)$ [1]. Extending this result to the difference spectrum encounters a possible obstruction in the case of non-metrizable groups, because the difference spectrum need not be closed [2]. The nature of that obstruction is discussed in this talk.

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WCG Banach algebras associated to locally compact groups

G. Schlichting

Let G be a locally compact group, and let X be any one of the Banach algebras $C^*(G)$, $C_0(G)$, $B(G)$ or $A(G)$. We characterize the property that X, as a Banach space, is weakly compactly generated (WCG) in terms of conditions on G and its dual space. This is joint work with E. Kaniuth and A.T. Lau.

Coretraction problems and homological dimensions of Banach algebras

Yu. V. Selivanov

According to a well-known theorem of Phillips [1], the subspace c_0 of all sequences tending to zero is non-complemented in the space c_b of all bounded sequences. This result and certain generalizations of it have been used in an essential way by Helemskii [2] and others in the study of homology characteristics of Banach algebras. In the first part of the talk we will review some of such generalizations of the Phillips theorem. Here we shall examine coretraction problems for diagonal maps of tensor products of Banach spaces and Banach modules.

The second part of the talk will concern some applications of the results of the first part to computing homological dimensions of Banach algebras and Banach modules over them. In particular, we shall define a wide class of commutative C^*-algebras of global dimension greater than 2. We will also discuss the so-called "additivity formulae" for homological dimensions of tensor products of Banach algebras and an application of these formulae to the problem of describing of the sets of values taken by the homological dimensions in the class of Banach function algebras.

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On strong generation of $B(\mathcal{H})$ by two commutative C^*-algebras

A. Sołtysiak

Let $\mathcal{H}$ be a complex Hilbert space and let $\mathcal{S}$ be a subset of the algebra $B(\mathcal{H})$ of all bounded linear operators on $\mathcal{H}$. We say that the algebra $B(\mathcal{H})$ is *strongly generated* by $\mathcal{S}$ if the smallest subalgebra of $B(\mathcal{H})$ closed in the strong operator topology and containing $\mathcal{S}$ coincides with $B(\mathcal{H})$. The first result on the strong generation of $B(\mathcal{H})$ was given by C. Davis in [2]. He proved, in the case when the Hilbert space $\mathcal{H}$ is separable, that the algebra $B(\mathcal{H})$ is strongly generated by two unitary operators. Later, E. Nordgren, M. Radjabalipour, H. Radjavi, and P. Rosenthal have shown [3] that two Hermitian operators strongly generate $B(\mathcal{H})$ which implies that $B(\mathcal{H})$ is singly generated as a von Neumann algebra. These results show that for a separable Hilbert space $\mathcal{H}$ the algebra $B(\mathcal{H})$ is strongly generated by two commutative C^*-algebras. In [4] W. Żelazko raised the following

Problem. Is the algebra $B(\mathcal{H})$ of all operators on a complex Hilbert space $\mathcal{H}$ always generated in the strong operator topology by two commutative C^*-algebras?

We show that the answer to this question is positive. Namely we prove the following

Theorem. [1] Let $\mathcal{H}$ be a complex Hilbert space. The algebra $B(\mathcal{H})$ is strongly generated by a single one-dimensional (orthogonal) projection and a commuting family of unitary operators with cardinality not exceeding $\dim \mathcal{H}$.

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Joint difference spectra

Jan D. Stegeman

For the study of sets of non-synthesis the *difference spectrum* of two ideals (e.g. of the largest and the smallest closed ideal with a certain set of non-synthesis as hull) is a useful instrument; cf. [1]. Likewise for the difference spectrum of a function with respect to an ideal (e.g. of a function in the largest ideal with respect to the smallest ideal).

For two functions f and g, will the difference spectrum of $f+g$ (w.r.t. some ideal) be the union of those of f and g? Of course not, in general, because the 'bad' behaviour of f may be partly cancelled by that of g. But we can ask, more generally, for linear combinations (finite or infinite) $\sum c_i f_i$ for which the difference spectrum is (the closure of) the union of the difference spectra.

Reasons for studying this problem will be given and some results will be discussed. We will see that (in the case under consideration) metrizability of the underlying group plays a role.

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Indecomposable derivations for some Banach algebras of power series

H. Steiniger

The main purpose of this article is to construct derivations from some Banach algebras of power series into modules such that the derivations are unbounded on the polynomials. These are called indecomposable derivations.

For the Beurling algebras $A_\alpha^+(\mathbb{D})$ consisting of certain holomorphic functions on the unit disc, our investigations result in some interesting properties about primary ideals. Hereby we are able to proof the following: for $\alpha \geq 1$, a derivation with finite-dimensional range is either bounded on the polynomials, or unbounded on every dense subalgebra; for $m \in \mathbb{N}$ and for $\alpha \in [m, m+1)$, every finite-dimensional extension of $A_\alpha^+(\mathbb{D})$ splits algebraically if and only if $\alpha \geq m + \frac{1}{m+1}$.

We apply the approach previously used for the Banach algebras $C^n[0,1]$ in [1] to the algebras $A_\alpha^+(\mathbb{D})$, obtaining various related and some different results.

In a second part, we construct a derivation from a radical Banach algebra of powers series, such that the derivation is unbounded on the polynomials. We use a method developed in [2]. We discuss some variants of the proof, including the construction of such a derivation which maps into the Volterra algebra.

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Finite rank Hankel operators on Hardy and Bergman spaces — their spectrum and unitary equivalence class

E. Strouse

I will discuss the characterization of finite rank Hankel operators of several types (normal, big, little, and intermediate). Then I will explain recent results of Peller, Treil, and Megretski concerning the characterization of the unitary equivalence of a self adjoint operator to a Hankel operator T in terms of certain conditions on the spectrum of T, and how these results can be explained in terms of finite rank operators and their symbols.

Elements in Banach algebras possessing strong partial approximate identities

Marc P. Thomas

Let s be a non-nilpotent element of a commutative radical Banach algebra. Two conditions on s which have often arisen are as follows:

Definition 1. We say that s has a bounded approximate identity (b.a.i.) if there exists a norm-bounded sequence $\{e_i\}$ in the radical algebra such that $\lim_{i\to\infty} se_i = s$.

Definition 3. We say that s has a partial approximate identity (p.a.i.) if there exists a sequence $\{e_i\}$ in the radical algebra such that $\lim_{i\to\infty} se_i = s$.

Clearly, (1) is a stronger condition than (3), and, since it implies factorization of s, is often more than sufficient. Condition (3) is adequate for some conclusions but seems to be too weak for others. We introduce a third condition and allow for the more general situation of an element s in a (possibly non-commutative) radical Banach algebra relative to the (closed commutative) subalgebra $C(C(s))$ which consists of all elements which commute with every element commuting with s.

Definition 2. We that s has a strong partial approximate identity (s.p.a.i.) if there exists a sequence $\{e_i\}$ in $C(C(s))$ such that both $\lim_{i\to\infty} se_i = s$, and $\lim_{i\to\infty} se_i^2 = s$.

It is clear that (1) is stronger than (2) and that (2) is stronger than (3). We give examples of elements which satisfy (2) but not (1) and prove the following theorem which is a special case of the unbounded Kleinecke-Shirokov conjecture.

Theorem 1. Let s be an element possessing a strong partial approximate identity in a (possibly non-commutative) radical Banach algebra with identity adjoined. If s is in the radical and D is a derivation from the algebra to itself such that Ds is not in the radical, then $[Ds, s] \neq 0$ (ie. s does not commute with Ds).

Joint spectral radius

Yu. V. Turovskii

Almost all results of this report are from an unpublished paper by Professor V. S. Shulman and myself. A Banach algebra A is called *finite quasinilpotent* if the joint spectral radius $\rho(M) = 0$ for all finite subsets M of A, where $\rho(M)$ is defined as $\inf_n \|M^n\|^{1/n}$, where $\|M\| = \sup\{\|a\| : a \in M\}$, $M^1 = M$, and $M^{n+1} = M^n M$.

In [1] it was proved that any algebra of compact quasinilpotent operators is finite quasinilpotent. We study a joint spectral radius in connection with some problems of triangularization and of the existence of invariant subspaces of bounded linear operators. The results are aplied to algebras with compact elements (and also to tensor products of algebras and algebras of multiplications).

In particular, we prove that any radical compact Banach algebra A has a nontrivial ideal J such that J and A/J are finite quasinilpotent.

Also, we show that if $K(X)$ is algebra of all compact operators on Banach space X and $F(X)$ is a uniform closure of all operators of finite rank on X then $K(X)/F(X)$ is finite quasinilpotent.

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Closability of partially defined derivations on Banach algebras

A. R. Villena

Closability of partially defined derivations on Banach algebras is a fundamental problem which appears in two different contexts; namely in the study of the time-development of quantum systems and also in the study of the continuity of everywhere defined derivations on nonassociative Banach algebras. On the other hand, the nilpotency of the separating subspace of an everywhere defined derivation on a Banach algebra is an intriguing question which remains still unsolved. We show that the separating subspace $S(D)$ of a derivation defined on a subalgebra B of a complex Banach algebra A satisfies $B[B \cap S(D)]B \subset \mathrm{Rad}_B(A)$ provided that $BAB \subset B$ and $\dim(\mathrm{Rad}_J(A) \cap \bigcap_{n=1}^{\infty} B^n) < \infty$, where $\mathrm{Rad}_B(A)$ and $\mathrm{Rad}_J(A)$ denote the Baer and the Jacobson radicals of A respectively. From this we deduce the closability of partially defined derivations on complex semiprime Banach algebras with appropriate domains and the continuity of Lie derivations of the skew elements of a centrally closed prime semisimple Banach algebra with linear involution.

The non-negativity of certain linking numbers

J. Wermer

This talk is on joint work with Herbert Alexander.

For X and Y disjoint compact, oriented submanifolds of $\mathbb{R}^k$, we write $L(X,Y)$ for the linking number of X and Y in the one-point compactification of $\mathbb{R}^k$ (see [1]).

Theorem. Let γ be an oriented, smooth curve in $\mathbb{C}^2$. A necessary and sufficient condition for the existence of an analytic variety having γ as a boundary in the sense of Stokes' theorem is that $L(\gamma, C) \geq 0$ for every non-singular, algebraic curve in $\mathbb{C}^2$ which does not meet γ

We conjecture that the analoguous result holds if γ is replaced by a compact, oriented, smooth 3-manifold M in $\mathbb{C}^3$, and C is a smooth, algebraic curve in $\mathbb{C}^3$ which misses M.

We have proved this when M is the graph in $\mathbb{C}^3$ of a smooth function defined on the boundary $\partial\Omega$ of a bounded convex domain Ω in $\mathbb{C}^2$ with $\partial\Omega$ smooth.

References

[1] H. Alexander, *Linking and holomorphic hulls*, J. Diff. Geom. **38** (1993), 151–160.

A Künneth formula for Banach algebras

M. C. White

The torus is a product of two circles. The topological Künneth formula [1] explains how the second (co-)homology group of the torus can be seen to be the tensor product of the first (co-)homology groups of the two circles. We can also see how the generator of $H_2(S^1 \times S^1)$ is the product of the two generators of $H_1(S^1)$, geometrically we easily see how two closed paths bound the area which generates $H_2(S^1 \times S^1)$. In general a Künneth formula relates the n-th (co-)homology of a product with a direct sum of tensor products of (co-)homology of the multiplicands [2]. There are a number of different Künneth type formulae with various hypotheses. As well as those for topological spaces there are also formulae for the (co-)homology of tensor products and joins of algebras: these exist in both pure algebra [2] and also for various Banach algebras [3], [4]. It should be observed that the Banach algebras for which these results have been proved (CSL-algebras) have cohomology groups which are related to those of an associated topological space. In the talk we shall briefly explain how the CSL-algebra case is proved and then indicate how this motivates the proof for more general Banach algebras.

References

[1] E. H. Spanier, *Algebraic Topology.*

[2] S. MacLane, *Homology.*

[3] F. Gilfeather and R. R. Smith, *Cohomology for operator algebras: Joins,.*

[4] S. Power, *Homology for Operator Algebras I : Spectral homology for reflexive algebras*, J. Functional Analysis **131** (1995), 29–53.

Factoring in finite codimensional ideals of group algebras

G. Willis

Let I be a finite codimensional ideal in the group algebra $L^1(G)$. Then $I = L + R$ where L is a closed, left ideal with a bounded right approximate identity and R is a closed, right ideal with a bounded left approximate identity. It follows that each element of I is a sum of 2 products in I.

If G is amenable, then I itself has a bounded approximate identity, as was shown in [1], and so every element of I is a product. However, if G is not amenable, then I does not have a bounded approximate identity and a decomposition of this type seems to be best possible. It suffices for automatic continuity arguments.

An outline of the proof of this decomposition of I will be presented. The proof relies on probabilistic ideas generalising the fact that, if G is a finite group and μ is a probability measure whose support equals G, then the sequence of powers of μ converges to the Haar measure on G.

References

[1] T. Liu, A. v. Rooij, J.-K.Wang, *Projections and approximate identities for ideals in group algebras*, Trans. Amer. Math.Soc. **175** (1973), 469–482.

On the powers and resolvents of linear operators

J. Zemánek

We intend to discuss some recent results and problems connecting the Kreiss resolvent condition with the behaviour of the powers and their various means.