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# Stochastics II

## 1. Tutorial

**Exercise 1 (5 Points)**

Mrs. Smith has made a cheesecake for her two daughters. Eating more than three quaters of the cake will give indigestion to anyone. While she is away, the older daughter eats a piece of the cake and the younger one takes a piece of what is left by her sister. We assume that the size of each of the two pieces eaten by the sisters is random and uniformly distributed over what is currently available. What is the expected size of the remaining piece given that neither daughter gets indigestion.

*Hint: The joint density of the random variable $(X, Y)$, where $X$ stands for the size of the piece eaten by the older sister and $Y$ for the size of the piece eaten by the younger one is given by*

$$f_{X,Y}(x, y) = \frac{1}{1-x} \mathbb{1}_{\{x,y\geq 0\}} \mathbb{1}_{\{x+y\leq 1\}}.$$

**Exercise 2 (2 + 2 + 1 Points)** Let $\Omega = [0, 1)$, $\mathcal{F}$ be the $\sigma$-algebra of Borel sets on the interval $[0, 1)$ and $P$ be the Lebesgue measure on $[0, 1)$. Furthermore let $\mathcal{G}_1$ be the $\sigma$-algebra, generated by the intervals

$$\left[0, \frac{1}{2}\right), \left[\frac{1}{2}, 1\right)$$

and $Y$ be a discrete random variable defined by

$$Y(x) = i \text{ für } x \in \left[\frac{i}{4}, \frac{i+1}{4}\right), i = 0, \ldots, 3.$$

Define the random variable

$$\forall x \in \Omega : \xi(x) = x^2$$

on $\Omega$ and calculate

(i) the conditional expectation $E(\xi|\mathcal{G}_1)$,

(ii) the conditional expectation $\eta = E(\xi|Y)$,

(iii) the conditional expectation $E(\eta|\mathcal{G}_1)$.

**Exercise 3 (5 Points)** Let $\Omega = [0, 1)$, $\mathcal{F}$ be the $\sigma$-algebra of Borel sets on the interval $[0, 1)$ and $P$ be the Lebesgue measure on $[0, 1)$. Find $E(\xi|\eta)$ if

$$\xi(x) = 2x^2 \text{ and } \eta(x) = \begin{cases} 2x & \text{if } x \in \left[0, \frac{1}{2}\right) \\ 2x - 1 & \text{if } x \in \left[\frac{1}{2}, 1\right). \end{cases}$$