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Stochastics II

10. Tutorial

Let $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,\infty)}, P)$ be the underlying filtered probability space for the whole assignment.

Exercise 1 (3+1 **Points)**

(i) Let τ be a stopping time. Show that $\mathcal{F}_\tau$ is a σ-field and that τ is measurable with respect to $\mathcal{F}_\tau$.

(ii) Show that for every $s \in [0,\infty]$ the equality $\mathcal{F}_\tau = \mathcal{F}_s$ holds for the stopping time $\tau :\equiv s$.

Exercise 2 (4 **Points)** Let τ be a optional time and $\mathcal{G}$ be the σ-field containing all the events $A \in \mathcal{F}_\infty$ for which $A \cap \{\tau \leq t\} \in \mathcal{F}_{t+}$ for every $t \geq 0$. Show that $\mathcal{G} = \mathcal{F}_{\tau+}$.

Exercise 3 (2+2+2 **Points)** Let σ and τ be stopping times.

(i) Show that $\mathcal{F}_{\sigma\wedge\tau} = \mathcal{F}_\sigma \cap \mathcal{F}_\tau$.

(ii) Show that

$$\{\sigma < \tau\},\ \{\sigma > \tau\},\ \{\sigma \leq \tau\},\ \{\sigma \geq \tau\},\ \{\sigma = \tau\}$$

are elements of $\mathcal{F}_\sigma \cap \mathcal{F}_\tau$.

(iii) Let Y be a $\mathbb{R}^D$-valued $\mathcal{F}_\sigma$-measurable random variable. Show that $\mathbb{1}_{\{\sigma\leq\tau\}}Y$ is $\mathcal{F}_{\sigma\wedge\tau}$-measurable.

Exercise 4 (3 **Points)** Assume we are in the situation of Example 8.14, but we omit the assumption that τ is bounded. Show that $(N_t^{(\varphi,\sigma,\tau)})_{t\in[0,\infty)}$ is a martingale.

Hint:You should rather use the statement of the Example 8.14, instead of modifying the corresponding calculations.