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Stochastics II

4. Tutorial

Exercise 1 (4+2 **Points**) Let $(\mathbb{R}, \mathcal{B}(\mathbb{R}^n), P)$ be a probability space, $B \in \mathcal{B}(\mathbb{R}^n)$ and $\varepsilon > 0$ arbitrary.

(i) Use the principle of appropriate sets to show that there exists a closed set F and an open set G such that $F \subset B \subset G$ and $P(G \setminus F) < \varepsilon$.

(ii) Show that there exists a compact subset $A \subset B$, such that $P(B \setminus A) \leq \varepsilon$.

Exercise 2 (4 **Points**) Let $X = (X_t)_{t\in[0,\infty)}$ and $Y = (Y_t)_{t\in[0,\infty)}$ be real-valued processes with P-a.s. rightcontinuous paths. Furthermore X and Y are modifications of each other. Show that X and Y are indistinguishable.

Exercise 3 (7 **Points**) Let $X := (X_t)_{t\in[0,\infty)}$ be a stochastic process and $(\mathcal{F}_t^X)_{t\in[0,\infty)}$ the filtration generated by X. Show that $X_t - X_s$ is independent of $\mathcal{F}_s^X$ for all $0 \leq s < t$ if and only if X has independent increments, i.e. the family $(X_{t_k} - X_{t_{k-1}})_{k=1,\ldots,n}$, where $X_{t_0} := 0$ is independent for all $n \in \mathbb{N}$ and $0 \leq t_1 < \cdots < t_n \in [0, \infty)$.