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Stochastics II

9. Tutorial

Exercise 1 (7 **Points)** We consider the following game: Player A enters the game with an inital capital of a Euro, Player B with b Euro ($a, b \in \mathbb{N}$). In each round of the game a fair coin is tossed. If the coin shows 'heads', Player A receives one Euro from Player B. Otherwise Player B gets one Euro from Player A. The game ends when one of the players runs out of money. The random time, at which the game ends is denoted by τ.

(i) Show that $\mathrm{E}[\tau] < \infty$.

(ii) Derive the probability that Player A wins the game.

(iii) Compute $\mathrm{E}[\tau]$.

Hint: Consider a Bernoulli(1/2) random walk $(X_n)_{n\in\mathbb{N}_0}$ as introduced in Example 2.5 and show that $M_n = X_n^2 - n$ is a martingale.

Exercise 2 (1+2+2 **Points)** Let $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,\infty)}, P)$ be the underlying filtered probability space.

(i) Show that every constant and non-negative real random variable is a stopping time.

(ii) Let $\sigma,\ \tau$ be stopping times. Show that $\sigma + \tau$ is a stopping time.

(iii) Let $\sigma > 0$ be a stopping time and τ be a optional time. Show that $\sigma + \tau$ is a stopping time.

Exercise 3 (4+2 **Points)** Let $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,\infty)}, P)$ be the underlying filtered probability space and let $(\tau_n)_{n\in\mathbb{N}}$ be a sequence of optional times.

(i) Show that

$$\sup_{n\in\mathbb{N}} \tau_n, \quad \inf_{n\in\mathbb{N}} \tau_n, \quad \limsup_{n\in\mathbb{N}} \tau_n, \quad \liminf_{n\in\mathbb{N}} \tau_n$$

are optional times.

(ii) Show that $\sup_{n\in\mathbb{N}} \tau_n$ and $\tau_1 \wedge \tau_2$ are stopping times, if every $\tau_n,\ n \in \mathbb{N}$, is a stopping time.