Algebraic Geometry and Commutative Algebra.
Summer term 2024
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## DES <br> SAARLANDES

## Exercise Sheet 3

Exercise 1 Let $V=V\left(x^{2}-y z, x z-x\right)$. Show that $V$ is a union of three irreducible components, describe them and find their prime ideals.

## Exercise 2

a) Let $A$ and $B$ be $k$-algebras. Define a multiplication on the tensor product $A \otimes B$ which turns it into a $k$-algebra.
b) Let $V, W$ be affine varieties over $K$. Show that the coordinate ring $A(V \times W)$ is isomorphic to $A(V) \otimes A(W)$ as a $K$-algebra.

Exercise 3 Let $X \subseteq \mathbb{A}^{n}$ and $Y \subseteq \mathbb{A}^{m}$ be affine varieties. Prove that the coordinate ring $A(X \times Y)$ of the product is an integral domain and hence that $X \times Y$ is irreducible as well.

Exercise 4 Let $A$ be a subset of a topological space $X$. Prove that $\operatorname{dim} A \leq \operatorname{dim} X$.

