Algebraic Geometry and Commutative Algebra.
Summer term 2024
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Exercise Sheet 3

Exercise 1 Let $V = V(x^2 - yz, xz - x)$. Show that V is a union of three irreducible components, describe them and find their prime ideals.

Exercise 2

a) Let A and B be k-algebras. Define a multiplication on the tensor product $A \otimes B$ which turns it into a k-algebra.

b) Let V, W be affine varieties over K. Show that the coordinate ring $A(V \times W)$ is isomorphic to $A(V) \otimes A(W)$ as a K-algebra.

Exercise 3 Let $X \subseteq \mathbb{A}^n$ and $Y \subseteq \mathbb{A}^m$ be affine varieties. Prove that the coordinate ring $A(X \times Y)$ of the product is an integral domain and hence that $X \times Y$ is irreducible as well.

Exercise 4 Let A be a subset of a topological space X. Prove that $\dim A \leq \dim X$.