Algebraic Geometry and Commutative Algebra.
Summer term 2024
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Exercise Sheet 7

Exercise 1 Show that the prevariety $\mathbb{P}^1$ constructed in the lecture is a variety.

Exercise 2 Show that any prevariety is a Noetherian topological space.

Exercise 3 Let X, Y be two prevarieties. Prove the following statements.

(a) If X, Y are two varieties, then $X \times Y$ is a variety.

(b) If X and Y are irreducible prevarieties, then $X \times Y$ is irreducible.

Exercise 4

(a) Let X be a topological space. Show that X is Hausdorff if and only if the diagonal Δ_X is closed.

(b) Show that the intersection of any two affine open subsets U, V, of a variety X, is again an affine open subset. Show that this fails if X is merely a prevariety.

Hint: For both parts consider $(U \times V) \cap \Delta_X$.

Exercise 5 A subset of $\mathbb{P}^n_k$ is called a projective subspace if it is the projectivization $\mathbb{P}(U)$ of a non-zero linear subspace $U \leq k^{n+1}$. Define $\dim \mathbb{P}(U) := \dim U - 1$. A line in $\mathbb{P}^n$ is a one dimensional projective subspace.
Let $V_1, V_2 \subseteq \mathbb{P}^n_k$ be two projective subspaces. Show that if $\dim V_1 + \dim V_2 \geq n$, then $V_1 \cap V_2 \neq \emptyset$. Conclude that any two distinct lines in $\mathbb{P}^2_k$ meet in exactly one point.