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Exercise Sheet 9

Exercise 1 Let $Y \subseteq \mathbb{P}^3$ be the image of the 3-uple embedding of

$$\mathbb{P}^1 \to \mathbb{P}^3, \quad [t_0 : t_1] \mapsto [t_0^3 : t_0^2 t_1 : t_0 t_1^2 : t_1^3].$$

It is called a twisted cubic curve. Consider the projection

$$\pi \colon \mathbb{P}^3 \to \mathbb{P}^2, [x_0 : x_1 : x_2 : x_3] \mapsto [x_0 : x_2 : x_3].$$

Show that the image $\pi(Y)$ is a cuspidal cubic curve in the plane, and find its equations.

Exercise 2 Let V be a projective variety. Let V^a be its affine cone. Show that $\dim V^a = \dim V + 1$.

Exercise 3 Let $G(2, 4) \subseteq \mathbb{P}^5$ be the Grassmanian of lines in $\mathbb{P}^3$ (or of planes in k^4). We denote the homogeneous Plücker coordinates $G(2, 4)$ in $\mathbb{P}^5$ by $x_{i,j}$ for $1 \leq i < j \leq 4$. Show:

(a) $G(2, 4) = Z(x_{1,2}x_{3,4} - x_{1,3}x_{2,4} + x_{1,4}x_{2,3})$.

(b) Let $L \subset \mathbb{P}^3$ be an arbitrary line. Show that the set of lines in $\mathbb{P}^3$ that intersect L, considered as a subset of $G(2, 4) \subset \mathbb{P}^5$, is the zero locus of a linear homogeneous polynomial.

Exercise 4 Show that the following sets are projective varieties:

(a) the incidence correspondence

$$\{(L, a) \in G(l, n) \times \mathbb{P}^{n-1} : L \subset \mathbb{P}^{n-1} \text{ an } l-1 \text{ dimensional linear subspace with } a \in L\}$$

(b) the join of two disjoint varieties $X, Y \subseteq \mathbb{P}^n$, i.e. the union in $\mathbb{P}^n$ of all lines intersecting X and Y. (Hint: Use that the image of a projective variety under a regular map is closed.)