Third Annual Meeting SFB-TRR 195: Abstracts

Saarbrücken, September 9-13, 2019

Mohamed Barakat (University of Siegen): On the generation of rank 3 simple matroids with an application to Terao's freeness conjecture

In this talk I describe a parallel algorithm for generating all non-isomorphic rank 3 simple matroids with a given multiplicity vector. We apply our implementation in the HPC version of GAP to generate all rank 3 simple matroids with at most 14 atoms and a splitting characteristic polynomial. We have stored the resulting matroids alongside with various useful invariants in a publicly available, ArangoDB-powered database. As a byproduct we show that the smallest divisionally free rank 3 arrangement which is not inductively free has 14 hyperplanes and exists in all characteristics distinct from 2 and 5. Another database query proves that Terao's freeness conjecture is true for rank 3 arrangements with 14 hyperplanes in any characteristic.

Janko Böhm (TU Kaiserslautern): Feynman integrals in high energy physics and geometry

Feynman diagrams describe interaction processes in high energy physics, with the associated Feynman integral yielding the probability of the interaction process. The concept of Feynman integrals has proven to be useful in many applications beyond physics.

Via integration-by-parts identities, the number of terms of an arbitrary Feynman integral associated to a particle interaction can be reduced dramatically. Based on computational commutative algebra, we develop an algorithm for finding all IBP identities for a given diagram, and describe an implementation using the Singular GPI-Space framework for massively parallel computations in computer algebra.

A mathematical application of Feynman integrals arises in the study of generating series of Gromov-Witten invariants and their tropical counterparts. The generating series can be expressed as a sum of Feynman integrals. Using tropical degenerations and floor diagram techniques, we give a first result for a higher-dimensional target space.

This is joint work with Dominik Bendle, Wolfram Decker, Alessandro Georgoudis, Christoph Goldner, Hannah Markwig, Franz-Josef Pfreundt, Mirko Rahn, Pascal Wasser, and Yang Zhang.

Wolfram Decker (TU Kaiserslautern): OSCAR - Where are we?

Claus Fieker (TU Kaiserslautern): ANTIC - A progress report

Johannes Flake (RWTH Aachen University): Deligne's interpolation categories and their monoidal centers

Deligne constructed monoidal categories which, in a certain precise sense, interpolate categories of representations for some families of finite groups, like the family of all symmetric groups. These interpolation categories have relatively simple combinatorial definitions, but interesting structures. I will explain the construction, known results on the structure, and joint work with Robert Laugwitz on the monoidal center of these categories.

Meinolf Geck (University of Stuttgart): Computing Green functions of finite groups of Lie type.

Green functions for finite groups of Lie type were introduced by Deligne and Lusztig in the 1970's, using cohomological methods. The computation of these functions is a crucial step in the more general programme of determining the whole character tables of those groups. We report on some recent progress, which essentially relies on computer algebra methods.

Tommy Hofmann (Saarland University): The conjugacy problem in $\mathrm{GL}(n, \mathbb{Z})$

We consider the problem of deciding whether two matrices are conjugate. If the coefficient ring is a field, this problem can be easily solved by using the Jordan normal form or the rational canonical form. For more general coefficient rings, the situation becomes increasingly challenging, both from a theoretical and a practical viewpoint. In this talk, we show how the conjugacy problem for integer matrices can be efficiently decided using techniques from group and number theory. This is joint work with Bettina Eick and Eamonn O'Brien.

Johannes Hoffmann (Saarland University): Noncommutative rational functions in OSCAR

The titular noncommutative rational functions are, as one might expect, the noncommutative analog of the classical rational functions. Subsequently, they play an important role in noncommutative analysis and also free probability theory. From an algebraic viewpoint, they form a division ring called the free (skew) field.

A well-known approach to working with noncommutative rational functions is to represent them (essentially) as the inverse of a matrix with polynomial entries that are at most of degree one. For application purposes it is crucial to find such representations where the size of the matrix is at least close to being minimal. In theory this can be done with (commutative) Gröbner bases, but the number of variables involved prevents these computations from being viable in practice. A way to overcome this is by employing techniques from linear algebra to reduce the complexity of the problem and only fall back on Gröbner bases as a last resort.

In this talk, I will give an introduction to noncommutative rational functions and how to work with them. In particular, I will report on the ongoing process of implementing them in the OSCAR framework by utilizing the Nemo and Singular subsystems.

Michael Joswig (TU Berlin): The tropical geometry of shortest path

We study parameterized versions of classical algorithms for computing shortest-path trees. This is most easily expressed in terms of tropical geometry. Applications include the enumeration of polytropes, i.e., ordinary convex polytopes which are also tropically convex, as well as shortest paths in traffic networks with variable link travel times. Joint work with Benjamin Schröter

Marek Kaluba (TU Berlin): Sum of squares computations and GroupRings.jl

will present the package GroupRings.jl and briefly talk about its aims and design principles. The package implements group rings with prescribed basis and supports fast multiplication (via incremental caching). Then I will show how this design leads to efficient sum of squares computation in group rings.

The packages GroupRings.jl and PropertyT.jl have been used to solve a problem in geometric group theory, namely to prove that $\mathrm{Aut}(F_n)$, the automorphism group of the free group has property (T).

An example computations using those packages can be seen under those links:
`https://nextjournal.com/kalmar/property-t-in-julia`
`https://nbviewer.jupyter.org/gist/kalmarek/5274299a94e002c6b266c258fb6a5203`

Lars Kastner (TU Berlin): Tropical compactification in polymake

Tropical compactification plays an important role in tropical homology, for computing the homology of cellular sheaves on the compactification of a polyhedral complex. Rabinoff described the tropical compactification in purely combinatorial terms. In this talk we will explore the algorithmic aspects of Rabinoff's construction. For many applications it suffices to know the Hasse diagram of the compactification, which can be computed by a variation of Ganter's algorithm. This requires mathematical knowledge about the faces of the compactification. Furthermore, we will present our implementation in polymake. This is joint work with Kristin Shaw (Oslo) and Anna-Lena Winz (Berlin).

Vladimir Lazić (Saarland University): Classification of projective varieties

I will give a panoramic overview of the Minimal Model Program: its goals, what has been done so far and what remains to be done.

Viktor Levandovskyy (RWTH Aachen University): Working within finitely presented algebras. Theory, algorithmics and implementation of Singular:Letterplace with applications

The major technology in addressing constructive computations within finitely presented algebras is based on Groebner bases. Considered very generally, the procedures need not necessarily terminate, so a computational paradigm has to be changed, what affects various aspects of algorithmics, implementation and applications. We present the most important developments, including those, who have been implemented for the first time in history. Applications from the TRR and beyond will be widely addressed as well.

Jean Michel (Paris Diderot University): A port of the Chevie GAP3 package to Julia

I will talk about advantages/problems encountered when trying to port the Chevie package to Julia and progress so far.

Mathias Schulze (TU Kaiserslautern): Polynomials associated to graphs, matroids, and configurations

Kirchhoff (Symanzik) polynomials are obtained from a graph as a sum of monomials corresponding to (non-)spanning trees. They are of particular importance in physics in the case of Feynman graphs. One can consider them as special cases of matroid (basis) polynomials, or of configuration polynomials. However the latter two generalizations differ in case of non-regular matroids. This more general point of view has the advantage that the classes of matroids and configurations are stable under additional operations such as duality and truncation. In addition there are important configuration polynomials, such as the second graph polynomial, which are not of Kirchhoff type. I will give an introduction to the topic and present new results relating the algebro-geometric structure of the singular locus of configuration hypersurfaces to the underlying matroid structure. Their proofs make essential use of matroid theory and, in particular, rely on duality.

Roland Speicher (Saarland University): Sharp bounds for sums associated to graphs of matrices

We provide a simple algorithm for finding the optimal upper bound for sums of products of matrix entries of the form

$$S_\pi(N) := \sum_{j_1,\dots,j_{2m}=1}^{N} t^1_{j_1 j_2} t^2_{j_3 j_4} \cdots t^m_{j_{2m-1} j_{2m}}$$

where some of the summation indices are constrained to be equal. The upper bound in the size N of the matrices is easily obtained from a graph G associated to the constraints π in the sum.

Moritz Weber (Saarland University): Quantum Symmetries and Computations

Quantum symmetry, understood in Woronowicz's framework, is a feature modeled via compact quantum groups. I will report on recent progress in the understanding of certain compact quantum groups of a matrix type. This involves computer based experiments, categorial points of views, representation theory and new quantum group products, arising from the SFB funded work with Daniel Gromada. I will also mention ongoing research on Deligne categories with Laura Maaen and Johannes Flake as well as future projects on quantum information theory, partially based on the work with Simon Schmidt.