UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG 6.1 – MATHEMATIK
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Additional notes supplementing the exercises for the lecture topology

Winter term 2017/18

Sheet 4 Deadline: -

Exercise 15

In fact, d_M is even Lipschitz continuous and in particular uniformly continuous.

Part d) tells us that metric spaces are normal, Part c) tells us that Urysohn's Lemma (respectively an even stronger version [which is (considered as proberty of the space) sometimes called perfect normality] holds in metric spaces. Note that we recovered the normality in part d) from part c) here which already hints at exercise 43.

Exercise 16

That a set X is countable can be interpreted as $|\mathbb{N}| = |X|$ (i.e. there is a bijection from $\mathbb{N}$ to X) and $|\mathbb{N}| \geq |X|$ (i.e. there is a surjection from $\mathbb{N}$ to X). I normally use the first version and also intended it for this exercise, but the second version makes the exercise much more interesting (since otherwise, not even X is open).

Exercise 17

For me, this exercise shows that continuity is a local property: If $X = \cup_{x \in X} U_x$ is an open cover of X by neighbourhoods U_x of all points $x \in$X, it tells us that a function is continuous if and only if it is continuous in every U_x that is if and only if it is locally continuous.

Also note that an analogue version is true for finite closed covers. This version is quite useful to establish the continuity even of simple functions as

$$f : \mathbb{R} \to \mathbb{R}, x \mapsto \begin{cases} x, & \text{falls } x \geq 0 \\ 0, & \text{falls } x < 0 \end{cases} = \begin{cases} x, & \text{falls } x \in [0, \infty) \\ 0, & \text{falls } x \in (-\infty, 0] \end{cases}.$$

Also not that by this exercise and its version for finite closed covers, one can glue continuous functions on open/finitely many closed sets together to obtain a continuous function on the whole of X as long as the individual functions coincide on the intersections.
