UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG 6.1 – MATHEMATIK
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Additional notes supplementing the exercises for the lecture topology

Winter term 2017/18

Sheet 5 Deadline: -

Exercise 20

This exercise mainly serves to show how strange arbitrary topological spaces can behave. Note that the solution of part b) is that all subsets of X are compact. Since not all subsets are closed in (X, τ), this shows that 4.10 b) (compact subspaces of Hausdorff spaces are closed) cannot be generalized arbitrarily. The same result also shows that the cofinal topology appearing in this exercise is not Hausdorff.

Exercise 21

As can already be seen in Exercise 23, this Exercise is a nice tool to define topologies since one does not have to care about arbitrary unions and can instead just define a topology by writing down its intended base (and checking the conditions here). Note that the proof is quite similar to the corresponding result for subbases in the lecture.

Exercise 22

This exercise basically tells you that subbases are 'enough' to describe a topology, i.e. that some important features (continuity, coarseness) depend only on a subbase of the considered topology.

Exerceise 23

In particular, there are first-countable topological spaces which are not metrizable. Also, taking into account 5.6c), there are firs-countable, separable topological spaces which are not second-countable.
