UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG 6.1 – MATHEMATIK
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Additional notes supplementing the exercises for the lecture topology
Winter term 2017/18

Sheet 8 Deadline: -

Exercise 34

In the literature, several Definitions of subnets are used. While part b) means that they are not equivalent, part d) shows that the situation is not as bad as it may sound at first glance. An AA-subnet need not be a subnet in our sense (sometimes also called a Willard subnet), but there is a subnet which has the same topological properties (i.e. limits, limit points) since their associated filters are the same.

I included part c) mainly to prove part d), not for its own sake.

Exercise 35

The quotient topology which is defined here has many useful applications e.g. gluing points in a topological space (that is, one uses the equivalence relation which identifies those two points) or idenitfying all points in some subset of a topological space with a single point. Also, the usual topology on the projective space is the quotient topology.

Exercise 36

What is shown in part a) is often called diagonalization trick and can be used in functional analysis at several points when dealing with sequences of sequences and their subseqeunces.

Exercise 37

The reverse implication does not hold, either. For examples of topological spaces which are sequentially compact but not compact, consider the right-order topology on $\mathbb{R}$ or the long line cf. Steen, Seebach: Counterexamples in topology, p. 74 for the first and Wikipedia 'Long Line' for the second.
