CP-Semigroups and Dilations - Subproduct Systems and Superproduct Systems: The Multi-Parameter Case and Beyond

Abstract:

"Dilation" is an attempt to understand the structure of an object, thought of as "small", by embedding it into a "big" and, hopefully, better behaved one. In classical operator theory, one "dilates" a contraction c on a Hilbert space to a (co-)isometry or a unitary w on a bigger Hilbert space. From the beginning it became very clear that, to have a useful dilation, it is not enough to to dilate the single contraction c to the single, whatsoever, w; we need a simultaneous dilation of the whole discrete one-parameter semigroup generated by c, in the sense that each (non-negative integer) power of c is dilated by the corresponding power of w.

In these two talks we report our latest results with Orr Shalit on the dilation of semigroups of completely positive maps on unital C* or von Neumann algebras to semigroups of endomorphisms. In the continuous one-paramter case (semigroups over the positive halfline), this stands for dilating an irreversible evolution to a (more) revesible one. When the algebra is B(H), the algebra of all bounded operators on e Hilbert space, then the discrete one-parameter case includes the topic of dilating so-called row-contractions isometrically. When passing to discrete multi-parameter semigroups, it includes the dilation of commuting tuples.

Our scope in the work with Orr Shalit is, starting from the well-know fact that in the one-parameter case dilations can conveniently be obtained and studied in terms of so-called product systems of C* or von Neumann correspondences, to examine to which point this can be pushed forward to the multi-parameter case, or even semigroups over more general (also non-abelian) monoids. Soon, one understands that one has to generalize product systems to subproduct systems emerging directly from CP-semigroup (known) and superproduct systems emerging directly from existence of a dilation (new). So, in our work we have to deal with a whole bunch of generalizations: From the one-parameter case(s) to more general indexing monoids; from one-parameter product systems to product systems over more general monoids; from product systems to super- and subproduct systems; and last but not least (actually we start with that), from so-called full (or module) dilations to dilations that live no longer in the same Morita equivalence class (as before was always implicitly assumed).

In the first talk we will set up the general theory, discuss the relation to more classical topics, and present "Bhat's example". The latter is an example for that practically everything that can go wrong in the discrete one-parameter (=one-mapping) case does go wrong. Necessarily, Bhat's example has to be a proper (=non-unital or non-Markov) CP-semigroup, because in the one-parameter Markov case, these things cannot happen. This illustrates that not even in the well-known (discrete or continuous) one-parameter case, everything is as understood as literature tries to make us believe.

In the second talk, we basically present our own (class of) example(s), obtained while proving that a dilation of a discrete (normal) two-parameter CP-semigroup (on a von Neumann algebra) always exists. (The proof requires a detailed analysis of certain subsets of the permutation groups. A side result is an iff-criterion for how to put together one-parameter product systems to multi-parameter product systems.) This examples illustrates that, now in the Markov case, everything that cannot go wrong in the one-parameter does go wrong in the two-parameter case.