Saarland University
FACULTY 6.1 – MATH
Prof. Dr. Jörg Eschmeier
M.Sc. Manuel Kany

Exercises for the lecture topology

Winter term 2019/2020

Sheet 12 Deadline: 01.21.2019

Exercise 45 (3 Points)

Let X be a Hausdorff topological space. Show the equivalence of:

(i) X is normal.

(ii) For any two disjoint non-empty closed sets $F, G \subset X$, there is a continuous function $f : X \to [0, 1]$ such that $f|_F \equiv 0$, $f|_G \equiv 1$.

(iii) For any two disjoint non-empty closed sets $F, G \subset X$, there is a continuous function $f : X \to \mathbb{R}$ such that $f|_F \equiv 0$, $f|_G \equiv 1$.

(iv) For each closed set $A \subset X$ and each continuous function $f : A \to [a, b]$ ($a, b \in \mathbb{R}$ with $a < b$), there is a continuous function $F : X \to [a, b]$ with $F|_A = f$.

(v) For each closed set $A \subset X$ and each continuous function $f : A \to \mathbb{R}$, there is a continuous function $F : X \to \mathbb{R}$ with $F|_A = f$.

Exercise 46 (4 Points)

Let X be a normal topological space and let $A \subset X$ be a closed set. Show that for each bounded continuous function $f : A \to \mathbb{C}$, there is a bounded continuous function $F : X \to \mathbb{C}$ with $F|_A = f$ and $\|f\|_A = \|F\|_X$.

Exercise 47 (4 Points)

Let X be a Hausdorff topological space such that, for each open cover $\mathcal{U} = (U_i)_{i=1}^n$ of X, there is a continuous partition of unity $(\theta_i)_{i=1}^n$ with respect to $\mathcal{U}$. Show that X is normal.

Exercise 48* (2*+2*=4* Points)

Let X be a locally compact Hausdorff space and let $K \subset U_1 \cup \cdots \cup U_n$ be an open cover of a compact set $K \subset X$. Show that:

(a) There are open sets $V_1, \ldots, V_n \subset X$ with compact closures $\bar{V}_i \subset X$ such that $K \subset V_1 \cup \cdots \cup V_n$ and $\bar{V}_i \subset U_i$ for $i = 1, \ldots, n$.

(b) Show that there are continuous functions $f_1, \ldots, f_n : X \to [0, 1]$ with $\operatorname{supp}(f_i) \subset U_i$ for $i = 1, \ldots, n$ and $\sum_{i=1}^n f_i(x) = 1$ for all $x \in K$ (*Hint : Use Exercise 42*).

(please turn over)

You can also find the exercise sheets on our homepage:

https://www.math.uni-sb.de/ag/eschmeier/lehre/WS1920/top/index.html