Saarland University
FACULTY 6.1 – MATH
Prof. Dr. Jörg Eschmeier
M.Sc. Manuel Kany

Exercises for the lecture topology

Winter term 2019/2020

Sheet 13 Deadline: 01.28.2019

A Hausdorff topological space X is called a topological m-manifold if the topology of X possesses a countable basis and if each point $x \in X$ possesses an open neighbourhood that is homeomorphic to an open subset of $\mathbb{R}^m$.

Exercise 49 (4 Points)

Let X be a compact topological m-manifold. Show that there is a number N such that there exists a topological embedding (=homeomorphism onto its range) $f : X \to \mathbb{R}^N$.

(Hint : Choose an open cover $(U_i)_{i=1}^n$ of X such that there are homeomorphisms $g_i : U_i \to V_i \subset \mathbb{R}^m$ and use a continuous partition of unity relative to $(U_i)_{i=1}^n$ to construct a topological embedding $f : X \to \mathbb{R}^{n+nm}$.)

Exercise 50 (4 Points)

Show that a compact Hausdorff space is metrizable if and only if it is second countable.

Subsets A, B of a topological space X are said to be separated if $A \cap \bar{B} = \emptyset = \bar{A} \cap B$.

Exercise 51 (4 Points)

Let $A, B \subset X$ be connected non-empty subsets of a topological space. Show that $A \cup B$ is connected if and only if A and B are not separated

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Exercise 52 (4 Points)

Let $A \subset X$ be a subset of a topological space X. Decide which of the following implications are true:

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\begin{aligned}
A \text{ is connected} &\Leftrightarrow \operatorname{Int}(A) \text{ is connected} \\
A \text{ is connected} &\Leftrightarrow \bar{A} \text{ is connected} \\
A \text{ is connected} &\Leftrightarrow \partial A \text{ is connected.}
\end{aligned}
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You can also find the exercise sheets on our homepage:

https://www.math.uni-sb.de/ag/eschmeier/lehre/WS1920/top/index.html